

THE APPLICATION OF A COMPUTER METHOD TO THE ANALYSIS OF  
A GRIDDED CIRCULAR CURVED FRAME

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by

Shyi-Jiun Leou

B. S., Taipei Institute of Technology,  
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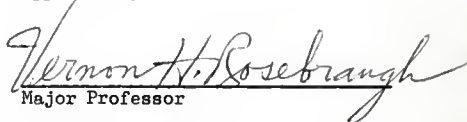
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Approved by:

  
Major Professor

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THE APPLICATION OF A COMPUTER METHOD TO THE ANALYSIS OF  
A GRIDDED CIRCULAR CURVED FRAME

By Shyi-Jiun Leou

SYNOPSIS

The analysis of the gridded circular curved frame can be simplified by using the computer method. Formulas are supplied by applying Castigliano's theorem. They show the relationship between the end moments, end torques and the corresponding rotations and displacements of the member end. A proposed matrix method associated with computer analysis is used in the procedure to solve the problem. The use of this method is illustrated by a detailed numerical example.

## INTRODUCTION

To meet the requirements of the modern trend in architecture and the clearance and space of some special structures, it seems that there will be an increasing use of gridded curved frameworks.

Because of the relative influence of deformation between curved girders and cross beams throughout the whole framework, the analysis of the gridded curved frame is a difficult task.

In this report, the computer method will be used to analyze such a framework. It allows the greatest possible simplification in the computation required. The member can have any curvatures in the plan, including segmental, but particular attention will be paid to the common circular-arc shape. Formulas are presented for frames of constant cross section that give the relation between the end moments, torques, and shears, and the corresponding rotations and displacements of the end sections. These relations have been derived from the expression for the total strain energy due to bending and torsion by applying Castigliano's theorem. From these relationships the stiffness matrix of individual members can be written and the stiffness matrix for the whole structure follows. Through computation with the electronic computer the end forces can be obtained. These end forces are combined with the fixed-end forces to obtain final end forces.

## STIFFNESS MATRIX OF AN ARC-SEGMENT

A beam curved in plan with end "a" free, and end "b" fixed, is shown in Fig. 1. Assume that the loading and translation are all perpendicular to the plane of the arc-segment.

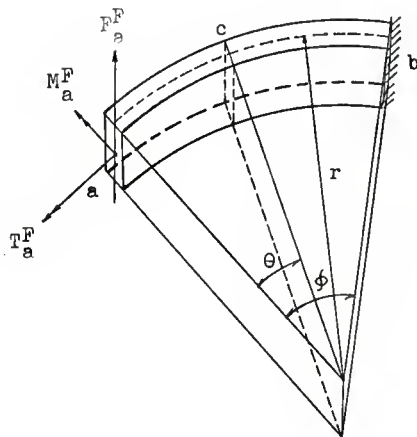


Fig. 1. A beam curved in plan with one fixed and one free end

At end "a" (Fig. 1), a moment  $M_a^F$ , a torque  $T_a^F$ , and a vertical force  $F_a^F$ , are acting on the beam as indicated by the vectors.

The properties of segment ab can be described by the partitioned matrix equation,

$$\begin{Bmatrix} P_a \\ \vdots \\ P_b \end{Bmatrix} = \begin{Bmatrix} K_{ab}^a & K_{ab}^b \\ \hline K_{ba}^a & K_{ba}^b \end{Bmatrix} \begin{Bmatrix} D_a \\ \vdots \\ D_b \end{Bmatrix}, \quad \text{---(1)}$$



where

$$P_a = \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad D_a = \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix},$$

$$P_b = \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \quad \text{and} \quad D_b = \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix}.$$

$D_{za}$  indicates displacement in the direction perpendicular to the plane at end "a".  $R_{ra}$  and  $R_{ta}$  indicate rotation about radius and tangential axes at end "a" respectively.

Then the K's in the stiffness matrix can be obtained as follows:

At any section an angular distance  $\theta$  from "a" the forces can be expressed as

$$\begin{cases} V_\theta = F_a^F, \\ M_\theta = F_a^F r \sin\theta + M_a^F \cos\theta - T_a^F \sin\theta, \\ T_\theta = F_a^F r (1 - \cos\theta) + M_a^F \sin\theta + T_a^F \cos\theta. \end{cases} \quad \text{-----}(2)$$

A sign convention that can be applied to the end of any span is shown in Fig. 2.

The moments are positive when, looking outwards from the center of curvature, they produce a clockwise rotation of the section.

The torques are positive when, looking along the tangent to the beam in a counterclockwise sense, they cause a clockwise rotation of the section.

The vertical forces are positive when directed upwards.



Fig. 2. A sign convention that can be applied to the end of any span

The total strain energy stored in the beam due to the acting forces, neglecting the effect of vertical shear, will be

$$\begin{aligned}
 U &= \int_0^\phi \frac{M_\theta^2 r d\theta}{2EI_r} + \int_0^\phi \frac{T_\theta^2 r d\theta}{2GJ} \\
 &= -\frac{r}{EI_r} \int_0^\phi \left[ \frac{M_\theta^2}{2} + \frac{KT_\theta^2}{2} \right] d\theta, \quad \text{-----}(3)
 \end{aligned}$$

where

$$K = -\frac{EI_r}{GJ}.$$

$EI_r$  and  $GJ$  are the flexible rigidity and torsional rigidity respectively.

The vertical displacement  $D_{za}$  at end "a" in the direction of  $F_a^F$  is, by Castigliano's theorem,  $D_{za} = \frac{\partial U}{\partial F_a^F}$ .

Then

$$\begin{aligned}
 \frac{\partial U}{\partial F_a^F} &= -\frac{r}{EI_r} \int_0^\phi \left[ F_a^F r^2 \sin^2 \theta + r M_a^F \sin \theta \cos \theta - r T_a^F \sin^2 \theta \right. \\
 &\quad \left. + K F_a^F r^2 (1 - \cos \theta)^2 + K r M_a^F \sin \theta - K r M_a^F \sin \theta \cos \theta \right. \\
 &\quad \left. + K r T_a^F \cos \theta - K r T_a^F \cos^2 \theta \right] d\theta.
 \end{aligned}$$

Integrate and get

$$D_{za} = -\frac{r^3}{4EI_r} F_a^F \left[ 2\phi - \sin 2\phi + K(6\phi + \sin 2\phi - 8 \sin \phi) \right] \\ + -\frac{r^2}{4EI_r} M_a^F \left[ 2 \sin^2 \phi + 2K(2 - 2\cos \phi - \sin^2 \phi) \right] \\ + -\frac{r^2}{4EI_r} T_a^F \left[ \sin 2\phi - 2\phi + K(4\sin \phi - 2\phi - \sin 2\phi) \right]. \text{-----}(4)$$

The rotation  $R_{ra}$  at "a" in the direction of  $M_a^F$  is  $R_{ra} = -\frac{\partial U}{\partial M_a^F}$ .

Then

$$-\frac{\partial U}{\partial M_a^F} = -\frac{r}{EI_r} \int_0^\phi \left[ F_a^F \sin \theta \cos \theta + M_a^F \cos^2 \theta - T_a^F \sin \theta \cos \theta + K F_a^F \sin \theta \right. \\ \left. - K F_a^F \sin \theta \cos \theta + K M_a^F \sin^2 \theta + K T_a^F \sin \theta \cos \theta \right] d\theta.$$

Integrate and get

$$R_{ra} = -\frac{r^2}{4EI_r} F_a^F \left[ 2 \sin^2 \phi + 2K(2 - 2\cos \phi - \sin^2 \phi) \right] \\ + -\frac{r}{4EI_r} M_a^F \left[ 2\phi + \sin 2\phi + K(2\phi - \sin 2\phi) \right] \\ + -\frac{r}{4EI_r} T_a^F \left[ 2(K-1)\sin^2 \phi \right]. \text{-----}(5)$$

The rotation  $R_{ta}$  at end "a" in the direction of  $T_a^F$  is  $R_{ta} = -\frac{\partial U}{\partial T_a^F}$ .

Then

$$-\frac{\partial U}{\partial T_a^F} = -\frac{r}{EI_r} \int_0^\phi \left[ -F_a^F \sin^2 \theta - M_a^F \sin \theta \cos \theta + T_a^F \sin^2 \theta + K F_a^F \cos \theta \right. \\ \left. - K F_a^F \cos^2 \theta + K M_a^F \sin \theta \cos \theta + K T_a^F \cos^2 \theta \right] d\theta.$$

Integrate and get

$$R_{ta} = -\frac{r^2}{4EI_r} F_a^F \left[ \sin 2\phi - 2\phi + K(4\sin \phi - 2\phi - \sin 2\phi) \right] + -\frac{r}{4EI_r} M_a^F \left[ 2(K-1)\sin^2 \phi \right] \\ + -\frac{r}{4EI_r} T_a^F \left[ 2\phi - \sin 2\phi + K(2\phi + \sin 2\phi) \right]. \text{-----}(6)$$

If the following substitutions are made in the above equations,

$$a_{11} = \frac{r^3}{4EI_r} \left[ 2\phi - \sin 2\phi + K(6\phi + \sin 2\phi - 8\sin \phi) \right].$$

$$a_{12} = \frac{r^2}{4EI_r} [2\sin^2\phi + 2K(2 - 2\cos\phi - \sin^2\phi)] = a_{21}.$$

$$a_{13} = \frac{r^2}{4EI_r} [\sin 2\phi - 2\phi + K(4\sin\phi - 2\phi - \sin 2\phi)] = a_{31}.$$

$$a_{22} = \frac{r}{4EI_r} [2\phi + \sin 2\phi + K(2\phi - \sin 2\phi)].$$

$$a_{23} = \frac{r}{4EI_r} [2(K-1)\sin\phi] = a_{32}.$$

$$a_{33} = \frac{r}{4EI_r} [2\phi - \sin 2\phi + K(2\phi + \sin 2\phi)].$$

Equation (4), (5) and (6) can now be expressed in the form of a matrix:

$$\begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial U}{\partial F_a^F} \\ -\frac{\partial U}{\partial M_a^F} \\ -\frac{\partial U}{\partial T_a^F} \end{Bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{----- (7)}$$

or

$$\begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix} = (A) \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{----- (8)}$$

Therefore

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = (A)^{-1} \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix}, \quad \text{----- (9)}$$

Equation (9) shows that  $(A)^{-1}$  is a transformation matrix between displacements and forces at end "a", but this is what the matrix  $K_{ab}^a$  in Eq. (1) stands for. So we get

$$K_{ab}^a = (A)^{-1}.$$

If we rewrite Eq. (2) in the matrix form, we get

$$\begin{Bmatrix} V_{\theta} \\ M_{\theta} \\ T_{\theta} \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r \sin \theta & \cos \theta & -\sin \theta \\ r(1-\cos \theta) & \sin \theta & \cos \theta \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}. \text{-----(10)}$$

Then the stresses at end "b" can be obtained by substituting the values of  $F_a^F$ ,  $M_a^F$  and  $T_a^F$  into Eq. (10) and by changing the angle  $\theta$  into  $\phi$ , or

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ r \sin \phi & \cos \phi & -\sin \phi \\ r(1-\cos \phi) & \sin \phi & \cos \phi \end{pmatrix} (A)^{-1} \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix}. \text{-----(11)}$$

Thus

$$K_{ba}^a = - \begin{pmatrix} 1 & 0 & 0 \\ r \sin \phi & \cos \phi & -\sin \phi \\ r(1-\cos \phi) & \sin \phi & \cos \phi \end{pmatrix} (A)^{-1}. \text{-----(12)}$$

To determine the values of  $K_{ba}^b$  and  $K_{ab}^b$ , we keep end "a" of the curved beam in Fig. 1. fixed, and end "b" free. Then a set of stress equations for the section at an angular distance  $\theta$  from end "b" can be obtained, and are

$$\begin{Bmatrix} V_{\theta} \\ M_{\theta} \\ T_{\theta} \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -r \sin \theta & \cos \theta & \sin \theta \\ r(1-\cos \theta) & -\sin \theta & \cos \theta \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}. \text{-----(13)}$$

Similarly, following the steps from Eq. (3) to Eq. (7), we get

$$\begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \text{-----(14)}$$

or

$$\begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix} = (B) \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}.$$

Therefore

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = (B)^{-1} \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix}. \text{-----}(15)$$

Each element of the B matrix in Eq. (14) is equal to each corresponding element of A matrix in Eq. (7), but the signs are not all the same. The elements  $b_{11}$ ,  $b_{13}$ ,  $b_{22}$ ,  $b_{31}$  and  $b_{33}$  have the same sign as the element  $a_{11}$ ,  $a_{13}$ ,  $a_{22}$ ,  $a_{31}$  and  $a_{33}$ , but  $b_{12}$ ,  $b_{21}$ ,  $b_{23}$  and  $b_{32}$  have the opposite sign to  $a_{12}$ ,  $a_{21}$ ,  $a_{23}$  and  $a_{32}$ .

Comparing Eq. (15) with Eq. (1), we know that

$$K_{ba}^b = (B)^{-1}. \text{-----}(16)$$

The stresses at end "a" can be obtained by substituting  $F_b^F$ ,  $M_b^F$ ,  $T_b^F$  into Eq. (13) and by changing the angle  $\theta$  into  $\phi$ .

So we get

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ -r \sin \phi & \cos \phi & \sin \phi \\ r(1-\cos \phi) & -\sin \phi & \cos \phi \end{pmatrix} (B)^{-1} \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix}. \text{-----}(17)$$

Thus

$$K_{ab}^b = - \begin{pmatrix} 1 & 0 & 0 \\ -r \sin \phi & \cos \phi & \sin \phi \\ r(1-\cos \phi) & -\sin \phi & \cos \phi \end{pmatrix} (B)^{-1}. \text{-----}(18)$$

As an example the stiffness matrix of a whole structure is presented as follows:

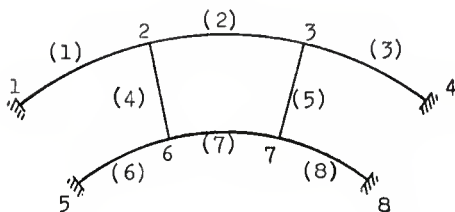


Fig. 3. Plan view of a frame with curved and straight members

Figure 3 indicates a frame with curved and straight members meeting at joints 1, 2, ..., 8. Numbers in parentheses in Fig. 3 indicate the numbering system for the bars. The relationship between member end forces and elastic member end displacements can be indicated by Eq.(19).

$$\{P\} = [K] \{D\} . \text{-----}(19)$$

Vector P in Eq. (19) represents member end forces. Its transpose is

$$\{P\}^T = [P_1^1, P_2^1, P_2^4, P_2^2, P_3^2, P_3^5, P_3^3, P_4^3, P_5^6, P_6^6, P_6^4, P_6^7, P_7^7, P_7^5, P_7^8, P_8^8] . \text{-----}(20)$$

Symbol  $P_1^1$  indicates the forces at end 1(the subscript) of member (1) (the superscript).

For each value of P on the right side of Eq. (20) there will be three components. For example,

$$P_3^2 = \begin{Bmatrix} F_3^2 \\ M_3^2 \\ T_3^2 \end{Bmatrix} . \text{-----}(21)$$

On the right side of Eq.(21) components indicate vertical force, moment and torque at end 3 of member (2).

Vector D in Eq.(19) represents member end displacements. Its transpose is

$$\{D\}^T = [D_1^1, D_2^1, D_2^4, D_2^2, D_3^2, D_3^5, D_3^3, D_4^3, D_5^6, D_6^6, D_6^4, D_6^7, D_7^7, D_7^5, D_7^8, D_8^8] \quad \text{-----}(22)$$

For each value of D on the right side of Eq.(22) there will be three components. For example,

$$D_2^4 = \left\{ \begin{matrix} D_{z2}^4 \\ R_{r2}^4 \\ R_{t2}^4 \end{matrix} \right\} \quad \text{-----}(23)$$

On the right side of Eq.(23), components indicate vertical displacement, rotation about the radius and rotation about the tangent, respectively, at end 2 of member (4).

The stiffness matrix K in Eq.(19) is an end displacement-end force transformation matrix. It is shown in Eq.(24) in which for example  $K_{23}^2$  is the stiffness matrix for member (2)(the superscript), and it relates the displacements at end 2(the first subscript) and the forces, which are induced by the displacements at end 2, at end 3(the second lower subscript).



$$[K] =$$

$P_1$	$D_1^1$	$D_2^1$	$D_2^4$	$D_2^2$	$D_3^2$	$D_3^5$	$D_3^3$	$D_4^3$	$D_5^6$	$D_6^6$	$D_6^4$	$D_6^7$	$D_7^7$	$D_7^5$	$D_7^8$	$D_8^8$
$P_1$	$K_{11}^1$	$K_{21}^1$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$P_2$	$K_{12}^1$	$K_{22}^1$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$P_4$	.	.	$K_{22}^4$	.	.	.	.	.	.	.	$K_{62}^4$	.	.	.	.	.
$P_2$	.	.	.	$K_{22}^2$	$K_{32}^2$	.	.	.	.	.	.	.	.	.	.	.
$P_2$	.	.	.	$K_{23}^2$	$K_{33}^2$	.	.	.	.	.	.	.	.	.	.	.
$P_5$	.	.	.	.	.	$K_{33}^5$	.	.	.	.	.	.	.	$K_{73}^5$	.	.
$P_3$	.	.	.	.	.	.	$K_{33}^3$	$K_{43}^3$	.	.	.	.	.	.	.	.
$P_3$	.	.	.	.	.	.	$K_{34}^3$	$K_{44}^3$	.	.	.	.	.	.	.	.
$P_4$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$P_5$	.	.	.	.	.	.	.	.	$K_{55}^6$	$K_{65}^6$	.	.	.	.	.	.
$P_6$	.	.	.	.	.	.	.	.	$K_{56}^6$	$K_{66}^6$	.	.	.	.	.	.
$P_6$	.	.	$K_{26}^4$	.	.	.	.	.	.	$K_{66}^4$	.	.	.	.	.	.
$P_7$	.	.	.	.	.	.	.	.	.	.	$K_{66}^7$	$K_{76}^7$	.	.	.	.
$P_7$	.	.	.	.	.	.	.	.	.	.	$K_{67}^7$	$K_{77}^7$	.	.	.	.
$P_5$	.	.	.	.	.	$K_{37}^5$	.	.	.	.	.	.	$K_{77}^5$	.	.	.
$P_8$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$K_{77}^8$	$K_{87}^8$
$P_8$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$K_{78}^8$	$K_{88}^8$

----- (24)

## FIXED-END STRESSES DUE TO UNIFORM LOAD

The end stresses for a beam curved in plan with both ends fixed and loaded uniformly with  $q$  per unit length of beam will now be determined.

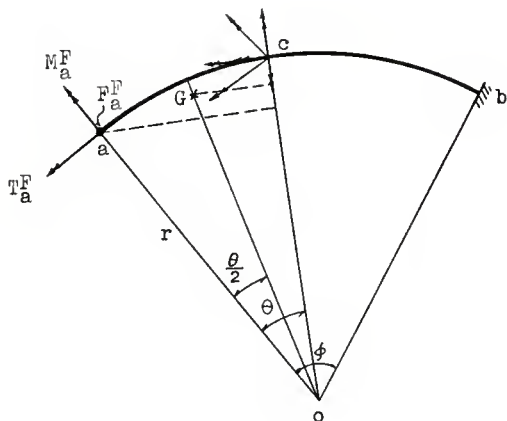


Fig. 4. Beam curved in plan with both ends fixed.

From Fig. 4, the vertical reaction  $F_a^F$  at end "a" is equal to

$$F_a^F = -\frac{qr\phi}{2},$$

and the distance OG from the center of curvature to the center of gravity of arc ac is

$$OG = \frac{r \sin \frac{\theta}{2}}{\frac{\theta}{2}}.$$

At a distance  $r\theta$  from end "a",

$$\begin{aligned}
 M_{\theta} &= M_a^F \cos\theta - T_a^F \sin\theta + F_a^F r \sin\theta - qr\theta \frac{r \sin\frac{\theta}{2}}{\frac{\theta}{2}} \sin\frac{\theta}{2} ; \\
 M_{\theta} &= M_a^F \cos\theta - T_a^F \sin\theta + \frac{qr^2\phi}{2} \sin\theta - qr^2(1-\cos\theta) ; \text{-----}(25) \\
 T_{\theta} &= M_a^F \sin\theta + T_a^F \cos\theta + F_a^F r(1-\cos\theta) - qr^2\theta(1 - \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \cos\frac{\theta}{2}) ; \\
 T_{\theta} &= M_a^F \sin\theta + T_a^F \cos\theta + \frac{qr^2\phi}{2}(1-\cos\theta) - qr^2(\theta - \sin\theta). \text{-----}(26)
 \end{aligned}$$

Applying a unit moment  $M_a^F = 1$  at "a". Then, on the section at an angular distance  $\theta$  from end "a",  $M_{\theta}^F = \cos\theta$  and  $T_{\theta} = \sin\theta$ .

As the beam is fixed at end "a",

$$\int_0^{\phi} M_{\theta}^F M_{\theta} \frac{rd\theta}{EI_r} + \int_0^{\phi} T_{\theta}^F T_{\theta} \frac{rd\theta}{GJ} = R_{ra} = 0 ,$$

or

$$\begin{aligned}
 &\int_0^{\phi} \frac{r}{EI_r} \left[ M_a^F \cos\theta - T_a^F \sin\theta + \frac{qr^2\phi}{2} \sin\theta - qr^2(1-\cos\theta) \right] \cos\theta d\theta \\
 &+ \int_0^{\phi} \frac{r}{GJ} \left[ M_a^F \sin\theta + T_a^F \cos\theta + \frac{qr^2\phi}{2}(1-\cos\theta) - qr^2(\theta - \sin\theta) \right] \sin\theta d\theta = 0.
 \end{aligned}$$

Integrate and get

$$\begin{aligned}
 &M_a^F \left[ \phi(K+1) - (K-1)\sin\phi\cos\phi \right] + T_a^F (K-1)\sin^2\phi + qr^2 \left[ \phi(K+1) + K\phi(\cos\phi + 1) \right. \\
 &\left. - \frac{1}{2}\phi(K-1)\sin^2\phi - 2(K+1)\sin\phi - (K-1)\sin\phi\cos\phi \right] = 0. \text{-----}(27)
 \end{aligned}$$

If a unit torque  $T_a^F = 1$  is applied at end "a", on the section at an angular distance  $\theta$  from end "a",  $M_{\theta}^F = \sin\theta$  and  $T_{\theta}^F = \cos\theta$ . As the beam is fixed at end "a",

$$\int_0^{\phi} M_{\theta}^F M_{\theta} \frac{rd\theta}{EI_r} + \int_0^{\phi} T_{\theta}^F T_{\theta} \frac{rd\theta}{GJ} = R_{ta} = 0 ,$$

or

$$\int_0^{\phi} \frac{r}{r_1} \left[ M_a^F \cos \theta - T_a^F \sin \theta + -\frac{qr^2}{2} \sin \theta - qr^2 (1 - \cos \theta) \right] (-\sin \theta) d\theta \\ + \int_0^{\phi} \frac{r}{r_2} \left[ M_a^F \sin \theta + T_a^F \cos \theta + -\frac{qr^2}{2} (1 - \cos \theta) - qr^2 (\theta - \sin \theta) \right] \cos \theta d\theta = 0.$$

Integrate and get

$$M_a^F (K-1) \sin^2 \phi + T_a^F \left[ \phi (K+1) + (K-1) \sin \phi \cos \phi \right] - qr^2 \left[ -\frac{1}{2} \phi^2 (K+1) \right. \\ \left. + -\frac{1}{2} \phi (K-1) \sin \phi \cos \phi - (K-1) \sin^2 \phi + K \phi \sin \phi - 2(K+1)(1 - \cos \phi) \right] = 0. \quad \text{-----}(28)$$

From Eq. (27) and (28), the values of  $M_a^F$  and  $T_a^F$  can be obtained for any known values of the uniform loading  $q$  and the angle  $\phi$ . It will be convenient to make the following substitutions in the above equations:

$$\begin{cases} A = \phi (K+1), \\ B = (K-1) \sin^2 \phi, \\ C = (K-1) \sin \phi \cos \phi, \\ D = \phi (K+1) + K \phi (\cos \phi + 1) - \frac{1}{2} \phi (K-1) \sin^2 \phi, \quad \text{-----}(29) \\ \quad - 2(K+1) \sin \phi - (K-1) \sin \phi \cos \phi, \\ E = -\frac{1}{2} \phi^2 (K+1) + -\frac{1}{2} \phi (K-1) \sin \phi \cos \phi - (K-1) \sin^2 \phi \\ \quad + K \phi \sin \phi - 2(K+1)(1 - \cos \phi). \end{cases}$$

Eq. (27) and (28) can now be expressed in the forms

$$(A-C)M_a^F + BT_a^F + qr^2 D = 0, \text{ and } \text{-----}(30)$$

$$BM_a^F + (A+C)T_a^F - qr^2 E = 0. \quad \text{-----}(31)$$

Solving for  $M_a^F$  and  $T_a^F$  gives

$$M_a^F = -\frac{qr^2 [D(A+C) + BE]}{A^2 - C^2 - B^2}, \text{ and } \text{-----}(32)$$

$$T_a^F = -\frac{qr^2 [E(A-C) + BD]}{A^2 - C^2 - B^2}. \quad \text{-----}(33)$$

## FIXED-END STRESSES DUE TO A CONCENTRATED LOAD

The end stresses for a beam curved in plan when both ends are fixed and loaded with a concentrated load located at an angular distance  $\alpha$  from end "a" will now be determined.

In Fig. 5, let the displacements at end "a" due to end stresses only

be  $\{D_{FNS}\} = \begin{Bmatrix} D_{za} \\ D_{ra} \\ D_{ta} \end{Bmatrix}$ , and let the displacements at end "a" due to the acting

concentrated load P only be  $\{D_P\}$ .

Then according to the fixed-end condition at "a", we get

$$\{D_{FNS}\} + \{D_P\} = 0. \quad \text{-----}(34)$$

By Eq.(8), we get

$$(A) \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -\{D_P\}, \quad \text{-----}(35)$$

or

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -(A)^{-1}\{D_P\}. \quad \text{-----}(36)$$

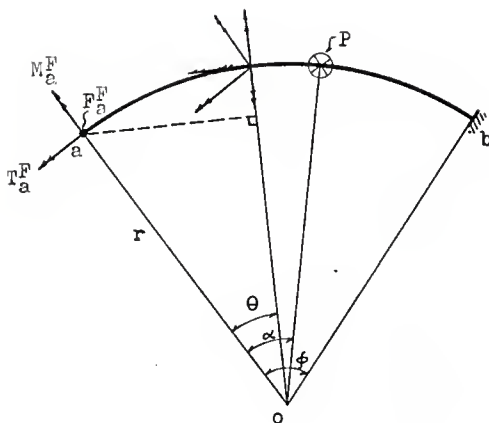


Fig. 5. Plan view of a curved beam with both ends fixed with a concentrated load  $P$  acting downward.  $0 \leq \theta \leq \alpha$

From Fig. 5, we find that the total stresses on the section at an angular distance  $\theta$  from end "a", when  $0 \leq \theta \leq \alpha$ , will be

$$\left. \begin{aligned} V_{\theta} &= F_a^F, \\ M_{\theta} &= M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta, \\ T_{\theta} &= M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta). \end{aligned} \right\} \text{-----} (37)$$

From Fig. 6, the total stresses on the section at an angular distance  $\theta$  from end "a", when  $\alpha \leq \theta \leq \phi$ , will be

$$\left. \begin{aligned} V_{\theta} &= F_a^F - P, \\ M_{\theta} &= M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta - Pr \sin(\theta - \alpha), \\ T_{\theta} &= M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) - Pr [1 - \cos(\theta - \alpha)]. \end{aligned} \right\} \text{-----} (38)$$

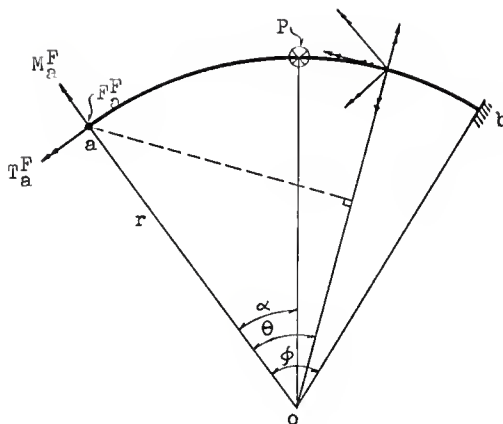


Fig. 6. Plan view of a curved beam with both ends fixed with a concentrated load  $P$  acting downward.  $\alpha \leq \theta \leq \phi$

If the effect of shear is neglected, the total strain energy is

$$U = \int_0^\alpha \frac{M_\theta^2}{2EI} r d\theta + \int_0^\alpha \frac{T_\theta^2}{2GJ} r d\theta + \int_\alpha^\phi \frac{M_\theta^2}{2EI} r d\theta + \int_\alpha^\phi \frac{T_\theta^2}{2GJ} r d\theta \quad \text{-----} (39)$$

Applying a unit force  $F_a^1 = 1$  at "a". Then, on the section at an angular distance  $\theta$  from the end "a",  $M_\theta^1 = r \sin \theta$  and  $T_\theta^1 = r(1 - \cos \theta)$  when  $0 \leq \theta \leq \phi$ . As the beam is fixed at end "a",

$$\begin{aligned} \frac{\partial U}{\partial F_a^1} = D_{za} &= \int_0^\alpha M_\theta^1 M_\theta \frac{rd\theta}{EI} + \int_0^\alpha T_\theta^1 T_\theta \frac{rd\theta}{GJ} \\ &+ \int_\alpha^\phi M_\theta^1 M_\theta \frac{rd\theta}{EI} + \int_\alpha^\phi T_\theta^1 T_\theta \frac{rd\theta}{GJ} = 0, \end{aligned}$$

or

$$\begin{aligned}
& \int_0^\alpha \frac{r^2}{EI_r} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta \right] \sin \theta d\theta \\
& + \int_0^\alpha \frac{r^2}{GJ} \left[ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) \right] (1 - \cos \theta) d\theta \\
& + \int_\alpha^\phi \frac{r^2}{EI_r} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta - Pr \sin(\theta - \alpha) \right] \sin \theta d\theta \\
& + \int_\alpha^\phi \frac{r^2}{GJ} \left[ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) - Pr (1 - \cos(\theta - \alpha)) \right] (1 - \cos \theta) d\theta = 0.
\end{aligned}$$

Collecting terms, we get

$$\begin{aligned}
& -\frac{r^2}{EI_r} F_a^F \left[ \int_0^\alpha (r \sin^2 \theta + Kr - 2Krcos\theta + Krcos^2 \theta) d\theta \right. \\
& \quad \left. + \int_\alpha^\phi (r \sin^2 \theta + Kr - 2Krcos\theta + Krcos^2 \theta) d\theta \right] \\
& + \frac{r^2}{EI_r} M_a^F \left[ \int_0^\alpha (\sin \theta \cos \theta + K \sin \theta - K \sin \theta \cos \theta) d\theta \right. \\
& \quad \left. + \int_\alpha^\phi (\sin \theta \cos \theta + K \sin \theta - K \sin \theta \cos \theta) d\theta \right] \\
& + \frac{r^2}{EI_r} T_a^F \left[ \int_0^\alpha (-\sin^2 \theta + K \cos \theta - K \cos^2 \theta) d\theta \right. \\
& \quad \left. + \int_\alpha^\phi (-\sin^2 \theta + K \cos \theta - K \cos^2 \theta) d\theta \right] \\
& = -\frac{r^2}{EI_r} \left[ \int_\alpha^\phi (-Pr \sin \theta \sin(\theta - \alpha) - KPr + KPr \cos(\theta - \alpha) \right. \\
& \quad \left. + KPr \cos \theta - KPr \cos(\theta - \alpha) \cos \theta d\theta \right] \dots \dots \dots (40)
\end{aligned}$$

Apply a unit moment  $M_a' = 1$  at "a". Then, on the section at an angular distance  $\theta$  from the end "a",  $M_\theta' = \cos \theta$  and  $T_\theta' = \sin \theta$  when  $0 \leq \theta \leq \phi$ .

As the beam is fixed at end "a",

$$\begin{aligned}
-\frac{\partial U}{\partial M_a^F} = R_{ra} &= \int_0^\alpha M_\theta' M_a^F \frac{rd\theta}{EI_r} + \int_0^\alpha T_\theta' T_a^F \frac{rd\theta}{GJ} \\
&+ \int_\alpha^\phi M_\theta' M_a^F \frac{rd\theta}{EI_r} + \int_\alpha^\phi T_\theta' T_a^F \frac{rd\theta}{GJ} = 0,
\end{aligned}$$



or

$$\begin{aligned}
 & \int_0^\alpha -\frac{r}{EI} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta \right] \cos \theta d\theta \\
 & + \int_0^\alpha -\frac{r}{GJ} \left[ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) \right] \sin \theta d\theta \\
 & + \int_\alpha^\phi -\frac{r}{EI} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta - Pr \sin(\theta - \alpha) \right] \cos \theta d\theta \\
 & + \int_\alpha^\phi -\frac{r}{GJ} \left\{ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) - Pr [1 - \cos(\theta - \alpha)] \right\} \sin \theta d\theta = 0.
 \end{aligned}$$

Collecting terms, we get

$$\begin{aligned}
 & -\frac{r}{EI} F_a^F \left[ \int_0^\alpha (r \sin \theta \cos \theta + K r \sin \theta - K r \sin \theta \cos \theta) d\theta \right. \\
 & \quad \left. + \int_\alpha^\phi (r \sin \theta \cos \theta + K r \sin \theta - K r \sin \theta \cos \theta) d\theta \right] \\
 & + -\frac{r}{EI} M_a^F \left[ \int_0^\alpha (\cos^2 \theta + K \sin^2 \theta) d\theta + \int_\alpha^\phi (\cos^2 \theta + K \sin^2 \theta) d\theta \right] \\
 & + -\frac{r}{EI} T_a^F \left[ \int_0^\alpha (-\sin \theta \cos \theta + K \sin \theta \cos \theta) d\theta + \int_\alpha^\phi (-\sin \theta \cos \theta + K \sin \theta \cos \theta) d\theta \right] \\
 & = -\frac{r}{EI} \int_\alpha^\phi \left[ -Pr \sin(\theta - \alpha) \cos \theta - K Pr \sin \theta + K Pr \cos(\theta - \alpha) \sin \theta \right] d\theta.
 \end{aligned}$$

----- (41)

Apply a unit torque  $T_a''' = 1$  at end "a". Then, on the section at an angular distance  $\theta$  from the end "a",  $M_\theta''' = -\sin \theta$  and  $T_\theta''' = \cos \theta$  when  $0 \leq \theta \leq \phi$ . As the beam is fixed at end "a",

$$\begin{aligned}
 -\frac{\partial U}{\partial T_a^F} = R_{ta} &= \int_0^\alpha M_\theta''' M_\theta - \frac{r d\theta}{EI} + \int_0^\alpha T_\theta''' T_\theta - \frac{r d\theta}{GJ} \\
 &+ \int_\alpha^\phi M_\theta''' M_\theta - \frac{r d\theta}{EI} + \int_\alpha^\phi T_\theta''' T_\theta - \frac{r d\theta}{GJ} = 0,
 \end{aligned}$$

or

$$\begin{aligned}
& \int_0^\alpha \frac{r}{EI_r} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta \right] (-\sin \theta) d\theta \\
& + \int_0^\alpha \frac{r}{GJ} \left[ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) \right] \cos \theta d\theta \\
& + \int_\alpha^\phi \frac{r}{EI_r} \left[ M_a^F \cos \theta - T_a^F \sin \theta + F_a^F r \sin \theta - Pr \sin(\theta - \alpha) \right] (-\sin \theta) d\theta \\
& + \int_\alpha^\phi \frac{r}{GJ} \left\{ M_a^F \sin \theta + T_a^F \cos \theta + F_a^F r (1 - \cos \theta) - Pr [1 - \cos(\theta - \alpha)] \right\} \cos \theta d\theta = 0.
\end{aligned}$$

Collecting terms, we get

$$\begin{aligned}
& -\frac{r}{EI_r} F_a^F \left[ \int_0^\alpha (-r \sin^2 \theta + Kr \cos \theta - Kr \cos^2 \theta) d\theta + \int_\alpha^\phi (r \sin^2 \theta + Kr \cos \theta - Kr \cos^2 \theta) d\theta \right] \\
& + \frac{r}{EI_r} M_a^F \left[ \int_0^\alpha (-\sin \theta \cos \theta + K \sin \theta \cos \theta) d\theta + \int_\alpha^\phi (-\sin \theta \cos \theta + K \sin \theta \cos \theta) d\theta \right] \\
& + \frac{r}{EI_r} T_a^F \left[ \int_0^\alpha (\sin^2 \theta + K \cos^2 \theta) d\theta + \int_\alpha^\phi (\sin^2 \theta + K \cos^2 \theta) d\theta \right] \\
& = -\frac{r}{EI_r} \int_\alpha^\phi \left[ Pr \sin(\theta - \alpha) \sin \theta - Pr \cos \theta + Pr \cos(\theta - \alpha) \cos \theta \right] d\theta.
\end{aligned}$$

-----(42)

Eq. (40), (41) and (42) can be written in a matrix form as

$$(Q) \quad \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = - \begin{Bmatrix} G \\ H \\ S \end{Bmatrix}, \quad \text{-----(43)}$$

where  $G, H, S^T$  corresponds to the vector  $D_p$  in Eq. (35).

Comparing Eq. (43) with Eq. (35), we know that

$$(Q) = (A). \quad \text{-----(44)}$$

So we get

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -(A)^{-1} \begin{Bmatrix} G \\ H \\ S \end{Bmatrix}, \quad \text{-----(45)}$$

where

$$\begin{aligned}
 G &= -\frac{r^2}{EI_r} \int_{\alpha}^{\phi} [-Pr \sin \theta \sin(\theta - \alpha) - KPr \\
 &\quad + KPr \cos(\theta - \alpha) + KPr \cos \theta - KPr \cos(\theta - \alpha) \cos \theta] d\theta, \\
 H &= -\frac{r}{EI_r} \int_{\alpha}^{\phi} [-Pr \sin(\theta - \alpha) \cos \theta - KPr \sin \theta + KPr \cos(\theta - \alpha) \sin \theta] d\theta, \\
 S &= -\frac{r}{EI_r} \int_{\alpha}^{\phi} [Pr \sin(\theta - \alpha) \sin \theta - Pr \cos \theta + Pr \cos(\theta - \alpha) \cos \theta] d\theta.
 \end{aligned}$$

Integrate and get

$$\begin{aligned}
 G &= -\frac{Pr^3}{4EI_r} \left\{ (1-K) \cos \alpha (\sin 2\phi - \sin 2\alpha) + (K-1) \sin \alpha (\cos 2\phi - \cos 2\alpha) \right. \\
 &\quad \left. - 2(\phi - \alpha) [(K+1) \cos \alpha + 2K] + 4K \sin(\phi - \alpha) + 4K(\sin \phi - \sin \alpha) \right\}, \quad \text{-----(46)}
 \end{aligned}$$

$$\begin{aligned}
 H &= -\frac{Pr^2}{4EI_r} [2(\phi - \alpha)(K+1) \sin \alpha + 4K(\cos \phi - \cos \alpha) - \cos \alpha (\cos 2\phi - \cos 2\alpha)(K-1) \\
 &\quad - \sin \alpha (\sin 2\phi - \sin 2\alpha)(K-1)], \quad \text{-----(47)}
 \end{aligned}$$

$$\begin{aligned}
 S &= -\frac{Pr^2}{4EI_r} [2(\phi - \alpha)(K+1) \cos \alpha - 4K(\sin \phi - \sin \alpha) + \cos \alpha (\sin 2\phi - \sin 2\alpha)(K-1) \\
 &\quad - \sin \alpha (\cos 2\phi - \cos 2\alpha)(K-1)]. \quad \text{-----(48)}
 \end{aligned}$$

Rewrite Eq.(38) and get

$$\begin{Bmatrix} V_{\theta} \\ M_{\theta} \\ T_{\theta} \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r \sin \theta & \cos \theta & -\sin \theta \\ r(1 - \cos \theta) & \sin \theta & \cos \theta \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} - \begin{Bmatrix} P \\ Pr \sin(\theta - \alpha) \\ Pr [1 - \cos(\theta - \alpha)] \end{Bmatrix} \quad \text{--(49)}$$

The fixed-end stresses at end "b" can be obtained by substituting the values of  $M_a^F$ ,  $T_a^F$ ,  $F_a^F$  into Eq.(49), and changing  $\theta$  to  $\phi$ . Thus we get

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r \sin \phi & \cos \phi & -\sin \phi \\ r(1 - \cos \phi) & \sin \phi & \cos \phi \end{pmatrix} (A)^{-1} \begin{Bmatrix} G \\ H \\ S \end{Bmatrix} + \begin{Bmatrix} P \\ Pr \sin(\phi - \alpha) \\ Pr [1 - \cos(\phi - \alpha)] \end{Bmatrix}. \quad \text{-----(50)}$$

## STIFFNESS MATRIX OF A STRAIGHT MEMBER

A straight beam with end "a" free and end "b" fixed is shown in Fig. 7. Assume that the loading and deformation are all in the Z direction.

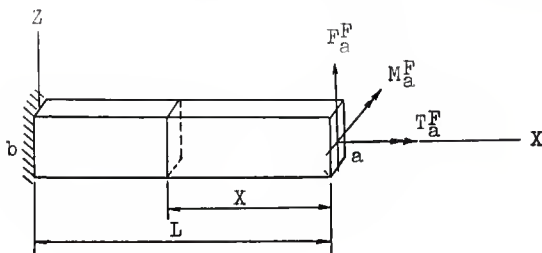


Fig. 7. Straight beam with one fixed and one free end

The properties of segment ab can be described by the partitioned matrix equation,

$$\begin{Bmatrix} P_a \\ P_b \end{Bmatrix} = \begin{bmatrix} \phi_{ab}^a & \phi_{ab}^b \\ \phi_{ba}^a & \phi_{ba}^b \end{bmatrix} \begin{Bmatrix} D_a \\ D_b \end{Bmatrix} \quad (51)$$

where

$$P_a = \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad D_a = \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix},$$

$$P_b = \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \quad \text{and} \quad D_b = \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix}.$$

$D_z$ ,  $R_y$  and  $R_x$  indicate the displacement in the Z direction and rotations about Y and X axis respectively.

Then the  $\phi$ 's in the stiffness matrix can be obtained as follows.

At any section of a distance x from end "a" the stresses can be expressed as

$$\begin{Bmatrix} V_x \\ M_x \\ T_x \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{-----(52)}$$

The total strain energy stored in the beam due to the acting forces, neglecting the effect of vertical shear, will be:

$$\begin{aligned} U &= \int_0^L \frac{M_x^2 dx}{2EI} + \int_0^L \frac{T_x^2 dx}{2GJ}, \\ &= \frac{1}{2EI} \left[ \int_0^L M_x^2 dx + K \int_0^L T_x^2 dx \right], \\ &= \frac{1}{2EI} \left\{ \int_0^L [(M_a^F)^2 - 2M_a^F F_a^F x + (F_a^F x)^2] dx + K \int_0^L (T_a^F)^2 dx \right\}, \\ &= \frac{1}{2EI} \left[ (M_a^F)^2 L - M_a^F F_a^F L^2 + \frac{1}{3} (F_a^F)^2 L^3 + KL (T_a^F)^2 \right]. \quad \text{-----(53)} \end{aligned}$$

By Castigliano's theorem

$$D_{za} = \frac{\partial U}{\partial F_a^F} = \frac{1}{2EI} \left[ -M_a^F L^2 + \frac{2}{3} L^3 F_a^F \right], \quad \text{-----(54)}$$

$$R_{ya} = \frac{\partial U}{\partial M_a^F} = \frac{1}{2EI} \left[ 2LM_a^F - L^2 F_a^F \right], \quad \text{-----(55)}$$

$$R_{xa} = \frac{\partial U}{\partial T_a^F} = \frac{1}{2EI} \cdot 2KL T_a^F. \quad \text{-----(56)}$$

Eq.(54), (55) and (56) can be expressed in the form of a matrix equation

$$\begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} = \begin{pmatrix} -\frac{L^3}{3EI} & -\frac{L^2}{2EI} & 0 \\ -\frac{L^2}{2EI} & -\frac{L}{EI} & 0 \\ 0 & 0 & \frac{L}{GJ} \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{-----(57)}$$

or

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} \quad \text{-----(58)}$$

So we get

$$\phi_{ab}^a = \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} \quad \text{-----(59)}$$

To get the stresses at end "b", substitute the value of Eq.(58) into Eq.(52) and let  $x=L$ . Then

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \phi_{ab}^a \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix}, \quad \text{-----(60)}$$

or

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix}.$$

So we get

$$\phi_{ba}^a = - \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} \quad \text{-----(61)}$$

To determine the values of  $\phi_{ba}^b$  and  $\phi_{ab}^b$ , we keep the end "a" of the beam ab in Fig. 7. fixed and let the end "b" free.

At any section of a distance  $x$  from end "b" the stresses can be expressed as in Eq.(62), or

$$\begin{cases} V_x = F_b^F, \\ M_x = M_b^F + F_b^F x, \\ T_x = T_b^F, \end{cases} \text{-----(62)}$$

or

$$\begin{Bmatrix} V_x \\ M_x \\ T_x \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \text{-----(63)}$$

Similarly following the steps from Eq.(53) to Eq.(57), we can get

$$\begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix} = \begin{pmatrix} -\frac{L^3}{3EI} & -\frac{L^2}{2EI} & 0 \\ -\frac{L^2}{2EI} & -\frac{L}{EI} & 0 \\ 0 & 0 & -\frac{L}{GJ} \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \text{-----(64)}$$

or

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix}. \text{-----(65)}$$

Thus

$$\phi_{ba}^b = \begin{pmatrix} -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix}. \text{-----(66)}$$

To get the stresses at end "a", substitute the value of Eq.(65) into Eq.(63) and let  $x=L$ , then

$$\begin{Bmatrix} P_a^R \\ M_a^R \\ T_a^R \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \phi_{ba}^b \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix} . \quad \text{-----}(67)$$

Thus

$$\phi_{ab}^b = - \begin{pmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{pmatrix} . \quad \text{-----}(68)$$



## NUMERICAL EXAMPLE

The gridded framework, with straight members and members curved in plan, shown in Fig. 8. is fixed at points 1,4,5 and 8. Points 2,3,6 and 7 are free to rotate and to translate perpendicular to the plane of the frame. The frame could be girders of a bridge. The cross sections A-A, B-B and C-C of Fig. 8. are shown in Fig. 9 and Fig. 10 respectively. The analysis of stresses will be separated into two parts. One is the calculation of stresses due to dead load and the other the calculation of stresses due to live load. When we do the calculation of stresses due to live load, the method of influence lines will be used in this report.

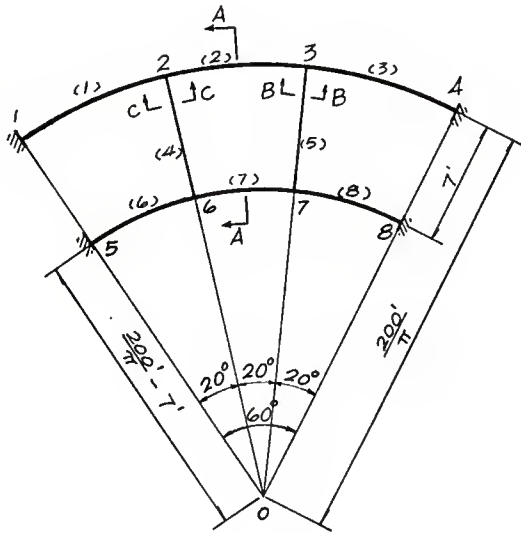


Fig. 8. Plan view of a frame with curved and straight members

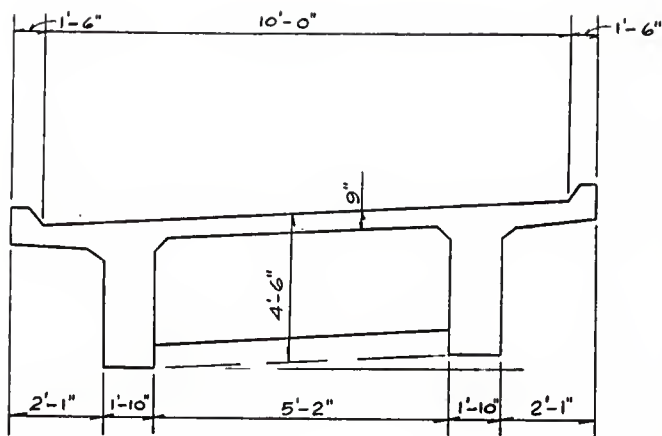


Fig. 9. Section A-A in Fig. 8.

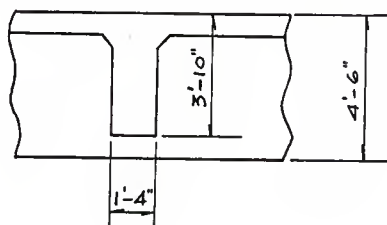


Fig. 10. Section B-B and C-C in Fig. 8.

## NOTATION AND ASSUMPTIONS

The following notation and assumptions are used in the calculations:

Modulus of elasticity of steel	$E_s = 29,000,000 \text{ lb/in}^2$
Ratio of $E_s$ to $E_c$	$n = 9$
Modulus of elasticity of concrete	$E_c = 3,220,000 \text{ lb/in}^2$
Moment of inertia of curved beams	$I_r = 863,370 \text{ in}^4$
Moment of inertia of straight beams	$I_t = 228,750 \text{ in}^4$
Uniform dead load for curved members	$q_c = 1.78 \text{ kips/ft}$
Uniform dead load for straight members	$q_s = 1.24 \text{ kips/ft}$

The torsion factor of curved beams is\*

$$J_c = \frac{b^3 d^3}{3.58(b^2 + d^2)} = \frac{(22)^3(54)^3}{3.58[(22)^2 + (54)^2]} = 137,000 \text{ in}^4.$$

The torsion factor of straight beams is

$$J_s = \frac{b^3 d^3}{3.58(b^2 + d^2)} = \frac{(16)^3(46)^3}{3.58[(16)^2 + (46)^2]} = 47,000 \text{ in}^4.$$

If

$$\frac{E}{G} = 2.25^*,$$

where  $E$  = modulus of elasticity in moment,

$G$  = modulus of elasticity in shear,

then the value of  $K$  for curved beams is

$$K = \frac{EI_r}{GJ_c} = 2.25 \frac{863370}{137000} = 14.2.$$

---

\* Refer to reference 11 in the Bibliography.

## DETERMINATION OF STRESSES DUE TO DEAD LOAD

The Slope-Deflection procedure in the computer method of stress analysis will be used in this report. First the fixed-end stresses will be determined in part A. Then these fixed-end stresses will be used to evaluate the member end stresses in part B.

## A. The calculation of the fixed-end stresses:

In this example we have  $\phi = -\frac{\pi}{3}$  and  $K = 14.2$  for curved members.

To evaluate the fixed-end moments and torques in Eq.(32) and (33) for each curved member, we shall use the electronic computer. FORGO source program 1 is used for this purpose and is shown on pages 61 and 62. Answers appear below on this page.

For the convenience of computer calculation, some substitutions have been made in the source program. The values BMF and TMF in the source program are used instead of the values  $M_a^F$  and  $T_a^F$  in Eq.(32) and (33). A,B,C,D, and E are the constants in Eq.(29). We shall use q as a positive value since we have considered that q is negative as we derived Eq.(32) and Eq.(33).

## ANSWERS OF PROGRAM 1

A	B	C	D	E
5.3058011	1.54410670	4.24239830	.01103570	-.01567320

BENDING-MOMENT  
-75.36911500

TORSION-MOMENT  
-.34668919

R  
63.66197600

A	B	C	D	E
5.30580110	1.54410670	4.24239830	.01103570	-.01567320

BENDING-MOMENT  
-59.10572400

TORSION-MOMENT  
-.27463982

R  
56.66197600

Now with the data given in the answers of program 1, the fixed-end stresses for each member can be listed.

In the list we use the notation  $M_{21}^F$  which will indicate the fixed-end moment of member (1) at end 2 (the first subscript refers to the end of a member and the second subscript refers to the number of bars) and the same subscription will be used for fixed-end torques and forces.

Because members (1), (2) and (3) have the same curvature and length, the fixed-end stresses due to uniform load should be the same. So we get

$$\begin{array}{ll} M_{11}^F = -75.369 \text{ ft-kips}, & M_{21}^F = +75.369 \text{ ft-kips}, \\ T_{11}^F = -0.347 \text{ ft-kips}, & T_{21}^F = -0.347 \text{ ft-kips}, \\ F_{11}^F = +19.778 \text{ kips}, & F_{21}^F = +19.778 \text{ kips}; \end{array}$$

$$\begin{array}{ll} M_{22}^F = -75.369 \text{ ft-kips}, & M_{32}^F = +75.369 \text{ ft-kips}, \\ T_{22}^F = -0.347 \text{ ft-kips}, & T_{32}^F = -0.347 \text{ ft-kips}, \\ F_{22}^F = +19.778 \text{ kips}, & F_{32}^F = +19.778 \text{ kips}; \end{array}$$

$$\begin{array}{ll} M_{33}^F = -75.369 \text{ ft-kips}, & M_{43}^F = +75.369 \text{ ft-kips}, \\ T_{33}^F = -0.347 \text{ ft-kips}, & T_{43}^F = -0.347 \text{ ft-kips}, \\ F_{33}^F = +19.778 \text{ kips}, & F_{43}^F = +19.778 \text{ kips}. \end{array}$$

Because members (6), (7) and (8) have the same curvature and length, the fixed-end stresses due to uniform load should be the same. So we get

$$\begin{array}{ll} M_{56}^F = -59.706 \text{ ft-kips}, & M_{66}^F = +59.706 \text{ ft-kips}, \\ T_{56}^F = -0.275 \text{ ft-kips}, & T_{66}^F = -0.275 \text{ ft-kips}, \\ F_{56}^F = +17.603 \text{ kips}, & F_{66}^F = +17.603 \text{ kips}; \end{array}$$

$$\begin{aligned}
 M_{67}^F &= - 59.706 \text{ ft-kips}, & M_{77}^F &= + 59.706 \text{ ft-kips}, \\
 T_{67}^F &= - 0.275 \text{ ft-kips}, & T_{77}^F &= - 0.275 \text{ ft-kips}, \\
 F_{67}^F &= + 17.603 \text{ kips}, & F_{77}^F &= + 17.603 \text{ kips}; \\
 \\ 
 M_{78}^F &= - 59.706 \text{ ft-kips}, & M_{88}^F &= + 59.706 \text{ ft-kips}, \\
 T_{78}^F &= - 0.275 \text{ ft-kips}, & T_{88}^F &= - 0.275 \text{ ft-kips}, \\
 F_{78}^F &= + 17.603 \text{ kips}, & F_{88}^F &= + 17.603 \text{ kips}.
 \end{aligned}$$

Because members (4) and (5) have the same length, the fixed-end stresses should be the same. So we get

$$\begin{aligned}
 M_{24}^F &= + 5.06 \text{ ft-kips}, & M_{64}^F &= - 5.06 \text{ ft-kips}, \\
 T_{24}^F &= 0, & T_{64}^F &= 0, \\
 F_{24}^F &= + 4.34 \text{ kips}, & F_{64}^F &= + 4.34 \text{ kips}; \\
 \\ 
 M_{35}^F &= + 5.06 \text{ ft-kips}, & M_{75}^F &= - 5.06 \text{ ft-kips}, \\
 T_{35}^F &= 0, & T_{75}^F &= 0, \\
 F_{35}^F &= + 4.34 \text{ kips}, & F_{75}^F &= + 4.34 \text{ kips}.
 \end{aligned}$$

B. Use the determined fixed-end stresses to evaluate the member end stresses.

This part of the calculation will be separated into 6 steps.

1. General approach.

Define the statics matrix  $A$  expressing the balancing forces  $\{Q\}$  at the joint in terms of the member end stresses  $\{P\}$ . Then by definition,

$$\{Q\} = [A] \{P\} . \text{-----}(69)$$

Define the geometry matrix  $B$  expressing values of elastic member end rotations and displacements  $\{E\}$  in terms of joint rotations and displacements  $\{X\}$ . Then by definition,

$$\{E\} = [B] \{X\} , \text{-----}(70)$$

where

$$[B] = [A]^T . \text{-----}(71)$$

Define the stiffness matrix  $K$  expressing the member end stresses  $\{P\}$  in terms of the elastic member end rotations and displacements  $\{E\}$ .

Then by definition,

$$\{P\} = [K] \{E\} . \text{-----}(72)$$

Substituting Eq.(70) and (71) into Eq.(72),

$$\{P\} = [K] [A]^T \{X\} = [G] \{X\} . \text{-----}(73)$$

Substituting Eq.(73) into Eq.(69),

$$\{Q\} = [A] [K] [A]^T \{X\} = [H] \{X\} . \text{-----}(74)$$

Therefore

$$\{X\} = [H]^{-1} \{Q\} . \text{-----}(75)$$

2. Draw the Q-X and P-E diagrams.

In Fig. 11, the Q-X diagram shows the joint balancing forces and corresponding deformations. All the forces are shown in the positive direction.

In Fig. 12, the P-E diagram shows the member end stresses and corresponding member end deformations. To keep the picture clear only the end stresses and corresponding deformations at end 2 of members (1), (2) and (4) are presented. Actually at each end of each member there will be three pairs of stress-deformation expressions. All the member end stresses are shown in the positive direction. Also, since members are perpendicular to each other at joints the torque of the straight bar is in the direction of the moment of the curved bar.

In Fig. 13, the diagram shows the equilibrium condition of balancing forces and end stresses at joint. According to this condition the sum of balancing forces and end stresses at a joint has to be equal to zero. Therefore, the balancing forces will be the sum of the end stresses at member ends. For example,

$$F_2 = F_2^1 + F_2^4 + F_2^2, \text{-----}(76)$$

$$M_2 = M_2^1 + T_2^4 + M_2^2, \text{-----}(77)$$

. . . . . ,

at joint 2 in Fig. 13.

It is noted that the values to which  $F_2^1, M_2^1, \dots$ , refer in Eq.(76) and (77) are the end stresses at the joints. They are not the member end stresses. The drawing of the equilibrium diagram is an important procedure by which we can establish matrix A in Eq.(69).



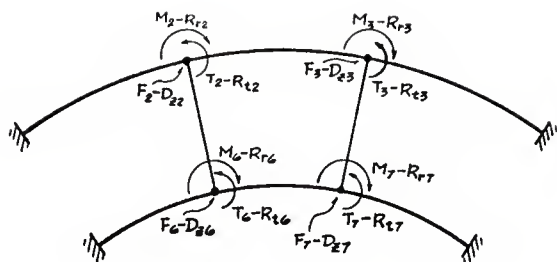


Fig. 11. Q-X diagram

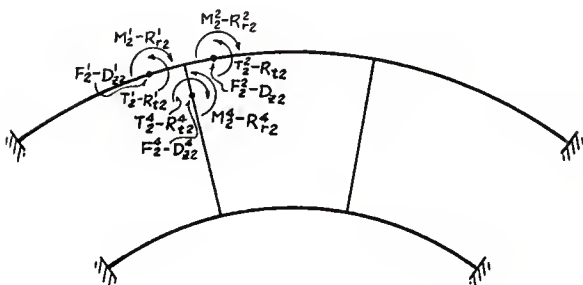
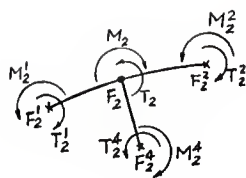
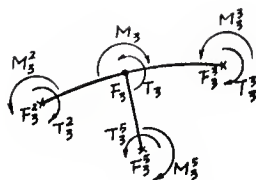


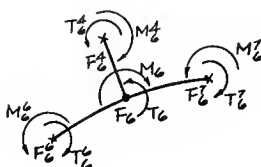
Fig. 12. P-E diagram



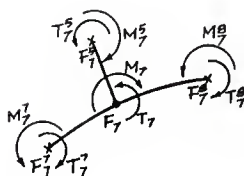
joint 2



joint 3



joint 6



joint 7

Fig. 13. Equilibrium diagrams of joints 2,3,6 and 7

### 3. Establish the matrices A, B and K.

To establish matrix A, we inspect every joint and write the equilibrium equations as in Eq.(76) and (77) for each balancing force component. Then matrix A can be written. For example, the entries of the first row in Eq. (78) are all zero, except the fourth, seventh and tenth which are equal to one. So we can write  $F_2 = F_2^1 + F_2^4 + F_2^2$ . This is also what Eq.(76) indicates. Matrix B is simply the transpose of matrix A.

If we partition the matrices A and B, we can rewrite them in the following form

$$[A] = [A_1 \mid A_2], \quad [B] = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

----- (7B)

	$F_1^1$	$M_1^1$	$T_1^1$	$F_2^1$	$M_2^1$	$T_2^1$	$F_2^4$	$M_2^4$	$T_2^4$	$F_2^2$	$M_2^2$	$T_2^2$	$F_3^2$	$M_3^2$	$T_3^2$	$F_3^5$	$M_3^5$	$T_3^5$	$F_3^3$	$M_3^3$	$T_3^3$	$F_4^3$	$M_4^3$	$T_4^3$
$F_2$	0.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$M_2$	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$T_2$	0.	0.	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$F_3$	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	1.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.
$M_3$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.
$T_3$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.
$F_6$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$M_6$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$T_6$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$F_7$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$M_7$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$T_7$	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$[A_1] =$

	$P_5^6$	$M_5^6$	$T_5^6$	$P_6^6$	$M_6^6$	$T_6^6$	$P_6^4$	$M_6^4$	$T_6^4$	$F_6^7$	$M_6^7$	$T_6^7$	$F_7^7$	$M_7^7$	$T_7^7$	$P_7^5$	$M_7^5$	$T_7^5$	$P_7^8$	$M_7^8$	$T_7^8$	$P_8^8$	$M_8^8$	$T_8^8$
$P_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$M_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$P_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$M_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$P_6$	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$M_6$	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$T_6$	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$P_7$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0
$M_7$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0
$T_7$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0

 $[A_2] =$ 

----- (79)

$$\begin{aligned}
 & \begin{matrix} D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} \end{matrix} \\
 [B_1] = & \begin{bmatrix}
 D_{z1}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r1}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t1}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 D_{z2}^1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r2}^1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t2}^1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 D_{z2}^4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r2}^4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t2}^4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 D_{z2}^2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r2}^2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t2}^2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 D_{z3}^2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r3}^2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t3}^2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 D_{z3}^5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r3}^5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 R_{t3}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 D_{z3}^3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r3}^3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t3}^3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 D_{z4}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{r4}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 R_{t4}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}
 \end{aligned}
 \tag{80}$$

$$\begin{aligned}
 & \begin{matrix} D_{z2} R_{r2} R_{t2} D_{z2} R_{r2} R_{t2} D_{z2} R_{r2} R_{t2} D_{z2} R_{r2} R_{t2} \\ D_{z5}^6 \\ R_{r5}^6 \\ R_{t5}^6 \\ D_{z6}^6 \\ R_{r6}^6 \\ R_{t6}^6 \\ D_{z6}^4 \\ R_{r6}^4 \\ R_{t6}^4 \\ D_{z6}^7 \\ R_{r6}^7 \\ R_{t6}^7 \\ D_{z7}^7 \\ R_{r7}^7 \\ R_{t7}^7 \\ D_{z7}^5 \\ R_{r7}^5 \\ R_{t7}^5 \\ D_{z7}^8 \\ R_{r7}^8 \\ R_{t7}^8 \\ D_{z8}^8 \\ R_{r8}^8 \\ R_{t8}^8 \end{matrix} \begin{bmatrix} \begin{matrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{matrix} \end{bmatrix} \\
& \left[ B_2 \right] = \text{-----} (81)
 \end{aligned}$$

$A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  all represent the matrices and they are shown in Eq.(78), (79), (80) and (81) respectively. It is noted that the notation of forces or deformations listed on the top and on the left side of the matrix is not included in the matrix. It is only for reference. For example, matrix A is a member end stress and balancing forces transformation matrix. So we list the notation of member end stresses on the top and balancing forces on the left side of the matrix to show to which pair of stresses and forces each entry of the matrix refers.

To establish the matrix K, we first have to determine the submatrices  $K_{11}^1$ ,  $K_{21}^1$ , . . . , etc. in Eq.(24).

The values of  $K_{11}^1$  and  $K_{55}^6$  are determined in program 2 which starts on page 62. Answers appear on page 43. In this example  $K_{11}^1 = K_{22}^2 = K_{33}^3$ , and  $K_{55}^6 = K_{66}^7 = K_{77}^8$ .

The values of  $K_{12}^1$  and  $K_{56}^6$  are determined in program 3 which starts on page 65. Answers appear on page 44. In this example  $K_{12}^1 = K_{23}^2 = K_{34}^3$ , and  $K_{56}^6 = K_{67}^7 = K_{78}^8$ .

The values of  $K_{22}^1$  and  $K_{66}^6$  are determined in program 4 which starts on page 67. Answers appear on page 44. In this example  $K_{22}^1 = K_{33}^2 = K_{44}^3$ , and  $K_{66}^6 = K_{77}^7 = K_{88}^8$ .

The values of  $K_{21}^1$  and  $K_{65}^6$  are determined in program 5 which starts on page 69. Answers appear on page 44. In this example  $K_{21}^1 = K_{32}^2 = K_{43}^3$ , and  $K_{65}^6 = K_{76}^7 = K_{87}^8$ .

The values of  $K_{22}^4$  and  $K_{26}^4$  are determined in program 6 which starts on page 71. Answers appear on page 45. In this example  $K_{22}^4 = K_{33}^5$ , and  $K_{26}^4 = K_{37}^5$ .

The values of  $K_{66}^4$  and  $K_{62}^4$  are determined in program 7 which starts on page 73. Answers appear on page 45. In this example  $K_{66}^4 = K_{77}^5$ , and

$$K_{62}^4 = K_{73}^5.$$

After getting these submatrices, we can insert them into Eq.(24) and get the matrix K.

For convenience in writing the source program, some changes have been made in the notations in the derived formulas as they are used in the source programs.

In program 2,  $\phi$ , K and  $[A]^{-1}$  in Eq.(9) have been changed into X, GK and [H], respectively.

In program 3,  $\phi$  and K in Eq.(12) have been changed into X and GK, respectively.

In program 4,  $\phi$ , K, [B] and  $[B]^{-1}$  in Eq.(16) have been changed into X, GK, [A] and [H], respectively.

In program 5,  $\phi$ , K and  $[B]^{-1}$  in Eq.(18) have been changed into X, GK and  $[A]^{-1}$ , respectively.

The answers of program 2-7 are listed as follows:

#### ANSWERS OF PROGRAM 2

STIFFNESS-MATRIX INV.(A) IN EQ.(9)

R

$$\begin{bmatrix} 20449.9000 & -227054.2800 & 14197.4900 \\ -227054.2800 & 3266355.0000 & -278175.0500 \\ 14197.4900 & -278175.0500 & 92871.4480 \end{bmatrix} = K_{11}^1$$

63.661976

STIFFNESS-MATRIX INV.(A) IN EQ.(9)

R

$$\begin{bmatrix} 29003.9020 & -286620.0200 & 17922.0810 \\ -286620.0300 & 3669879.6000 & -312540.7200 \\ 17922.0810 & -312540.7200 & 104344.7600 \end{bmatrix} = K_{55}^6$$

56.661976



## ANSWERS OF PROGRAM 3

STIFFNESS-MATRIX IN EQ.(12)

$$\begin{pmatrix} -20449.9000 & 227054.2800 & -14197.4900 \\ -227052.5100 & 1779295.9000 & -15968.6430 \\ -14197.2000 & 15966.6400 & -46637.3990 \end{pmatrix} = K_{12}^1$$

R

63.661976

STIFFNESS-MATRIX IN EQ.(12)

$$\begin{pmatrix} -29003.9020 & 286620.0200 & -17922.0810 \\ -286617.7800 & 1999109.6000 & -17941.4300 \\ -17921.7130 & 17939.1100 & -52398.9710 \end{pmatrix} = K_{56}^6$$

R

56.661976

## ANSWERS OF PROGRAM 4

STIFFNESS-MATRIX IN EQ.(16)

$$\begin{pmatrix} 20449.9000 & 227054.2800 & 14197.4900 \\ 227052.5100 & 3266355.0000 & 278175.0500 \\ 14197.4900 & 278175.0500 & 92871.4480 \end{pmatrix} = K_{22}^1$$

R

63.661976

STIFFNESS-MATRIX IN EQ.(16)

$$\begin{pmatrix} 29003.9020 & 286620.0200 & 17922.0810 \\ 286620.0300 & 3669879.6000 & 312540.7200 \\ 17922.0810 & 312540.7200 & 104344.7600 \end{pmatrix} = K_{66}^6$$

R

56.661976

## ANSWERS OF PROGRAM 5

STIFFNESS-MATRIX IN EQ.(18)

$$\begin{pmatrix} -20449.9000 & -227054.2800 & -14197.4900 \\ -227052.5100 & 1779295.9000 & 15968.6430 \\ -14197.2000 & -15966.6400 & -46637.3990 \end{pmatrix} = K_{21}^1$$

R

63.661976

STIFFNESS-MATRIX IN EQ.(18)

$$\begin{pmatrix} -29003.9020 & -286620.0200 & -17922.0810 \\ 286617.7800 & 1999109.6000 & 17941.4300 \\ -17921.7130 & -17939.1100 & -52398.9710 \end{pmatrix} = K_{65}^6$$

R

56.661976

## ANSWERS OF PROGRAM 6

$$\begin{bmatrix} 178954.0800 & 626339.2900 & 0.0000 \\ 626339.2900 & 2922916.7000 & 0.0000 \\ 0.0000 & 0.0000 & 66728.3960 \end{bmatrix} = K_{22}^4$$

$$\begin{bmatrix} -178954.0800 & -626339.2900 & -0.0000 \\ 626339.2900 & 1461458.3000 & -0.0000 \\ -0.0000 & -0.0000 & -66728.3960 \end{bmatrix} = K_{26}^4$$

## ANSWERS OF PROGRAM 7

$$\begin{bmatrix} 178954.0800 & -626339.2900 & 0.0000 \\ -626339.2900 & 2922916.7000 & 0.0000 \\ 0.0000 & 0.0000 & 66728.3960 \end{bmatrix} = K_{66}^4$$

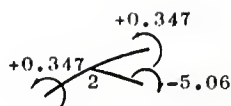
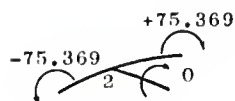
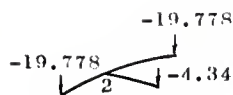
$$\begin{bmatrix} -178954.0800 & 626339.2900 & -0.0000 \\ -626339.2900 & 1461458.3000 & -0.0000 \\ -0.0000 & -0.0000 & -66728.3960 \end{bmatrix} = K_{62}^4$$

4. Calculate matrices G and  $H^{-1}$ .

Through matrix multiplication and inversion, the matrices G and  $H^{-1}$  can be easily evaluated. Programs 8 and 9 shown on pages 74 and 75 are examples of matrix multiplication and inversion.

## 5. Determine the vector Q.

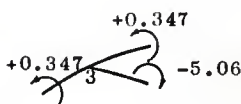
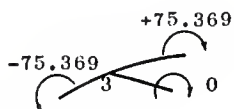
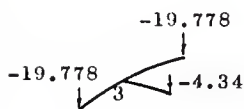
Using the fixed-end stresses at member ends listed on pages 32 and 33, vector Q can be determined from information shown in Fig. 14.



$$F_2 = -43.896 \text{ kips}$$

$$M_2 = 0$$

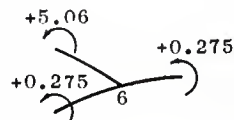
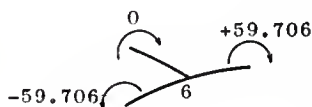
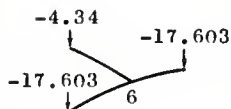
$$T_2 = -4.366 \text{ ft-kips}$$



$$F_3 = -43.896 \text{ kips}$$

$$M_3 = 0$$

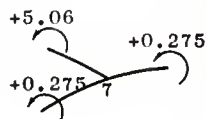
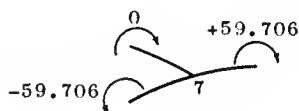
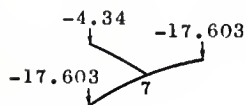
$$T_3 = -4.366 \text{ ft-kips}$$



$$F_6 = -39.546 \text{ kips}$$

$$M_6 = 0$$

$$T_6 = +5.61 \text{ ft-kips}$$



$$F_7 = -39.546 \text{ kips}$$

$$M_7 = 0$$

$$T_7 = +5.61 \text{ ft-kips}$$

Fig. 14. Balancing forces at joints 2, 3, 6 and 7

Therefore the vector Q is

$$\{Q\} = \begin{Bmatrix} F_2 \\ M_2 \\ T_2 \\ F_3 \\ M_3 \\ T_3 \\ F_6 \\ M_6 \\ T_6 \\ F_7 \\ M_7 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} -43.896 \\ 0.0 \\ -4.366 \\ -43.896 \\ 0.0 \\ -4.366 \\ -39.546 \\ 0.0 \\ +5.61 \\ -39.546 \\ 0.0 \\ +5.61 \end{Bmatrix}.$$

6. Calculate the member end stresses as indicated in vector P in Eq.(73).

Substitute the value of matrix  $H^{-1}$  and vector Q into Eq.(75). Vector X can then be determined. Then substitute the value of matrix G and vector X into Eq.(73). The vector P can be thus determined.

Here we have used a source program of matrix times vector. It is shown in program 10 on page 76.

Vector P was partitioned in the form of  $\{P\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$ .

Combining the fixed-end stresses with the member end stresses, vector P, we get the final member end stresses which are listed in Tables 1 and 2 on pages 85 and 86.

## DRAW INFLUENCE LINE

In this section, we shall draw the influence line for each stress component of each section shown in Fig. 15. If more sections were to be taken the influence lines would be smoother. However, we will define the ordinates only at sections shown in Fig. 15 when we draw the influence lines later. The changes of values between sections will be assumed linear.

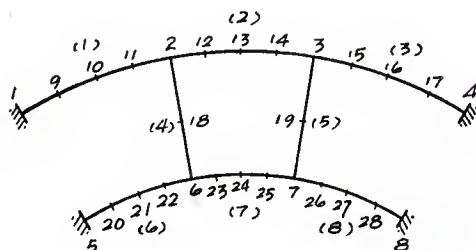


Fig. 15. Plan view of a framework with curved and straight members

First the fixed-end stresses will be determined in part A. Secondly the influence line will be drawn in part B.

#### A. The calculation of the fixed-end stresses.

Before determining the fixed-end stresses, the constants  $Q$ ,  $H$  and  $S$  in Eq.(46), (47) and (48) were evaluated in program 11 starting on page 77. Answers appear on page 49. Then substituting the values of  $Q$ ,  $H$  and  $S$  into Eq.(45), we get the fixed-end stresses at end "a". They are determined by program 12 starting on page 80. Similarly if the values of  $Q$ ,  $H$  and  $S$  are

substituted into Eq.(50) the fixed-end stresses at end "b" can be determined. This is performed by use of program 13 starting on page 82. Answers of program 12 and 13 appear on page 50 and 51.

## ANSWERS OF PROGRAM 11

	R	63.66197600	
G	H	S	Z
- .000140844240	- .000009378848	- .000006671969	.08726646
R	63.66197600		
G	H	S	Z
- .000066033581	- .000003845325	- .000001600150	.17453293
R	63.66197600		
G	H	S	Z
- .000017110796	- .000000864254	- .000000061282	.26179939
R	56.66197600		
G	H	S	Z
- .000099305638	- .000007429725	- .000005285392	.08726646
R	56.66197600		
G	H	S	Z
- .000046558573	- .000003046185	- .000001267605	.17453293
R	56.66197600		
G	H	S	Z
- .000012064381	- .000000684644	- .000000048546	.26179939
STOP END OF PROGRAM AT STATEMENT 0060 + 00 LINES			

## ANSWERS OF PROGRAM 12

R     63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.84546821	-3.20061630	.01030862	.08726646
R     63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.50000071	-2.87813180	.01644585	.17453293
R     63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.15455155	-1.07916620	.00820781	.26179939
R     56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.84546832	-2.84868920	.00917506	.08726646
R     56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.50000081	-2.56166610	.01463761	.17453293
R     56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.15455148	-.96050480	.00730520	.26179939
STOP    END OF PROGRAM AT    STATEMENT 0050 + 00 LINES			

## ANSWERS OF PROGRAM 13

R      63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.15453180	1.07909800	.00822890	.08726646
R      63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.49999930	2.87811500	.01644997	.17453293
R      63.66197600			
FIXED-END STRESSES			
F	M	T	Z
.84544850	3.20024310	.01027509	.26179939
R      56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.15453170	.96044500	.00732360	.08726646
R      56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.49991920	2.56164840	.01464101	.17453293
R      56.66197600			
FIXED-END STRESSES			
F	M	T	Z
.84544860	2.84835840	.00914525	.26179939
STOP   END OF PROGRAM AT   STATEMENT 0060 + 00 LINES			



### B. Draw the influence line.

This will be separated into four steps.

#### 1. Determine the vector Q.

Using the fixed-end stresses determined in programs 12 and 13, we can follow the same procedure to determine the vector Q under a unit load as we determined the vector Q under uniform load on page 46. For every acting position of the unit load, there will be one corresponding vector Q. Results are listed in Table 3-8 starting on page 87.

#### 2. Calculate final member end stresses.

Substituting the vector Q into Eq.(75), we get values of vector X. Then from Eq.(73), we get values of vector P, the member end stresses. Combining the member end stresses with the fixed-end stresses, we get the final member end stresses. They are listed in Table 9-12 starting on page 90.

#### 3. Calculate the stresses at a section between joints.

The stresses at a section between joints can be determined by substituting final member end stresses into Eq.(10), (13) and (52). Results are listed in Tables 13, 14, 15 and 16, in which the notation referring to each stress component is as it is defined earlier in the report. For example,  $F_{18}$  simply represents the vertical force at section 18 shown in Fig. 15. However here we did not use any number of bar as the subscript with the notation as in Table 13-16 starting on page 98.

#### 4. Draw the influence line.

When we draw the influence line we indicate only the ordinate above

the beam. Although data are available to draw an influence for each stress component of each section, we will draw only the influence line for section 13 as an example. For the convenience of drawing the influence line all the curved beams have been lengthened and have become imaginary straight beams.

Figures 16, 17 and 18 show the influence lines of vertical force, moment and torsion at section 13, respectively. The values of coordinates of these lines can be found from Tables 13, 14, 15 and 16. For example, to find coordinates of the influence line for vertical force at section 13, we turn to page 98 where is the Table 13. In the left most column of this table, we find  $F_{13}$ . Then values appearing in the same row with  $F_{13}$  are vertical forces at section 13 due to a unit load acting at each different section in turn.

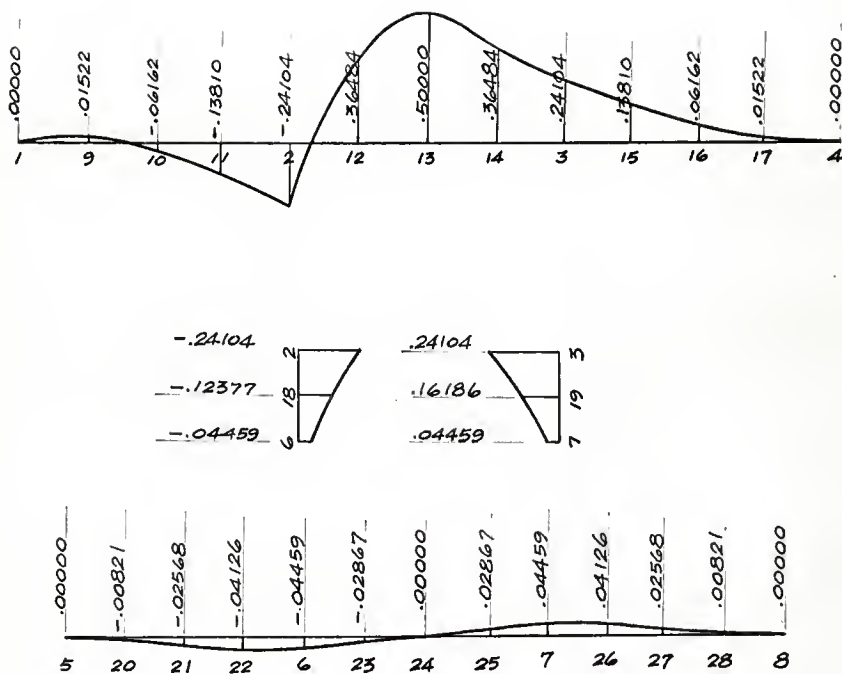


Fig. 16. Influence line for vertical force at section 13

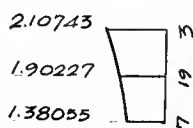
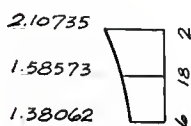
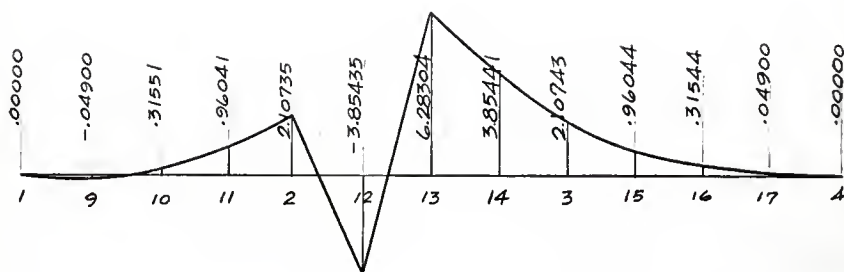


Fig. 17. Influence line for moment at section 13

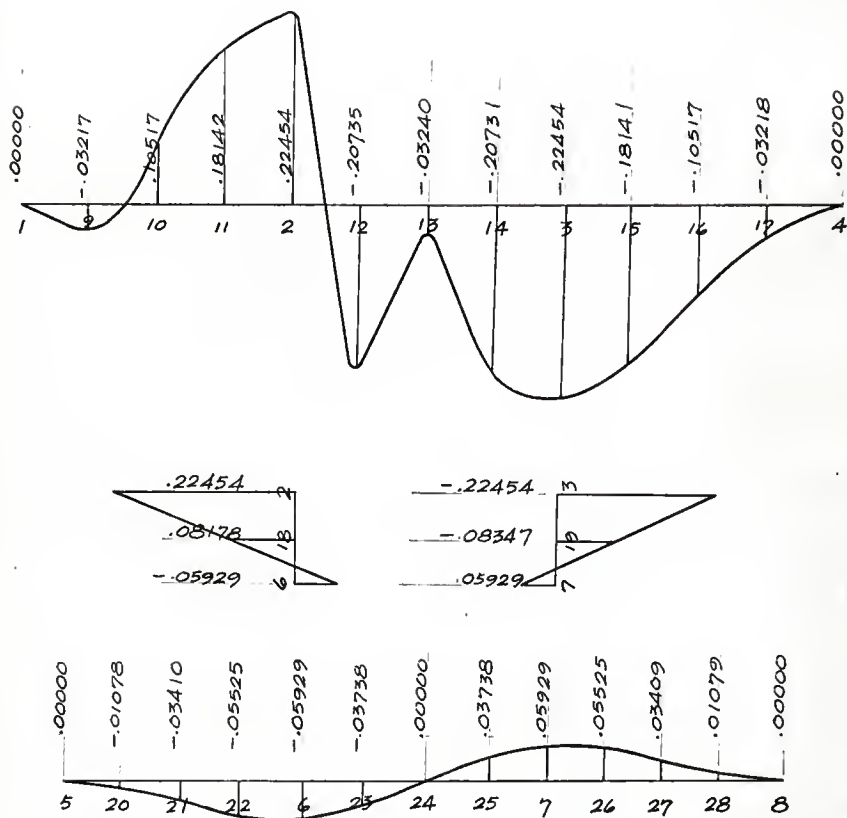


Fig. 18. Influence line for torsion at section 13

## CONCLUSION

All the formulas derived in this report are intended for general application to any kind of structure with members which are straight or curved in plan. Any more complicated structure can be analysed by the same procedure. In case the number of unknowns in the stiffness matrix exceeds the capacity of the memory storage of the computer available, the partitioning method is suggested. However if the members of a structure do not meet orthogonally the analysis of stresses will become more complicated.

## ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Professor Vernon H. Rosebraugh for his advice and guidance throughout this study.

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## APPENDIX

## APPENDIX

FORGO SOURCE PROGRAM.

```

C C PROGRAM 1 LFCU 11/5/66
C CAL. OF FIXED-END MOMENTS OF A CURVED-BEAM UNDER UNIF. LOAD
PI=3.1415926536
X=PI/9.
GK=14.2
A=X*(GK+1.)
B=(GK-1.)*SINF(X)*SINF(X)
C=(GK-1.)*SINF(X)*COSF(X)
D=A+GK*X*(COSF(X)+1.)-B*X/2.-2.*(GK+1.)*SINF(X)-C
E=(A+C)*X/2.-B+GK*X*SINF(X)-2.*(GK+1.)*(1.-COSF(X))
Q=1.78
I=1
R=200./PI
10 BN=-Q*R**2*(D*(A+C)+B*E)
TN=+Q*R**2*(E*(A-C)+B*D)
CD=A**2-B**2-C**2
BMF=BN/CD
TMF=TN/CD
15 FORMAT(/6X,1HA,11X,1HB,11X,1HC,11X,1HD,11X,1HE)
PUNCH 15
16 FORMAT(5F12.8)
PUNCH 16,A,B,C,D,F
17 FORMAT(/5X,14HBENDING-MOMENT,7X,14HTORSION-MOMENT,13X,1HR/)

```

```

PUNCH 17
18 FORMAT(3F20.8)
PUNCH 18,RMF,TMF,R
IF(I-2)20,30,30
20 R=R-7.
I=I+1
GO TO 10
30 STOP
END

```

```

C C PROGRAM 2 LECU 12/4/66
C CAL. OF STIFFNESS MATRIX INV.(A) IN EQ.(9)
DIMENSION A(10,10),S(10),T(10),H(10,10)
INDEX=1
PI=3.1415926536
X=PI/9.
EI=3220000.*863370./144000.
R=200./PI
READ,N
5 Q=R/(4.*EI)
SR=R*Q
CR=R*SR
GR=14.2
X2=2.*X
S2=SINF(X2)

```

```

S1=SINF(X)
SS=S1**2
C1=COSEF(X)
A(1,1)=CR*(X2-S2+GK*(6.*X+S2-8.*S1))
A(1,2)=SR*(2.*SS+2.*GK*(2.-2.*C1-SS))
A(2,1)=A(1,2)
A(1,3)=SR*(S2-X2+GK*(4.*S1-X2-S2))
A(3,1)=A(1,3)
A(2,2)=Q*(X2+S2+GK*(X2-S2))
A(2,3)=Q*(2.*(GK-1.)*SS)
A(3,2)=A(2,3)
A(3,3)=Q*(X2-S2+GK*(X2+S2))
NN=N-1
DO 6 I=1,3
DO 6 J=1,3
6 H(I,J)=A(I,J)
H(1,1)=1./H(1,1)
DO 110 M=1,NN
K=M+1
50 DO 60 I=1,M
S(I)=0
DO 60 J=1,M
60 S(I)=S(I)+H(I,J)*H(J,K)
D=0
DO 70 I=1,M
70 D=D+H(K,I)*S(I)
D=-D+H(K,K)

```

```

      H(K,K)=1./D
      DO 80 I=1,M
80    H(I,K)=-S(I)*H(K,K)
      DO 90 J=1,M
      T(J)=0
      DO 90 I=1,M
90    T(J)=T(J)+H(K,I)*H(I,J)
      DO 100 J=1,M
100   H(K,J)=-T(J)*H(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
110   H(I,J)=H(I,J)-S(I)*H(K,J)
      15 FORMAT(/5X,34HSTIFFNESS-MATRIX INV.(A) IN EQ. 9.,19X,1HR//)
      PUNCH 15
160   PUNCH 8,R
      8 FORMAT(52X,F12.6)
170   PUNCH 9,((H(I,J),J=1,N),I=1,N)
      9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
25    INDEX=INDEX+1
      R=R-7.
      GO TO 5
30    STOP
      END
C      DATA OF PROGRAM 2

```

```

C C PROGRAM 3 LFCU 12/4/66
C CAL. OF STIFFNESS MATRIX IN EQ.(12)
  DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3)
  INDFX=1
  PI=3.1415926536
  X=PI/9.
  R=200./PI
101 READ 1,INA,IMA,IMB
  1 FORMAT(3I8)
102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)
  48 FORMAT(3F16.4)
  5 S1=SINF(X)
  C1=COSE(X)
  B(1,1)=-1.0
  B(1,2)=-0.
  B(1,3)=-0.
  B(2,1)=-R*S1
  B(2,2)=-C1
  B(2,3)=+S1
  B(3,1)=-R*(1.-C1)
  B(3,2)=-S1
  B(3,3)=-C1
  DO 121 I=1,3
  DO 121 J=1,3
121 BQ(I,J)=B(I,J)
  DO 7 I=1,INA

```

```

      DO 7 J=1,IMB
      SUM=0.
      DO 6 K=1,IMA
      6 SUM=RQ(I,K)*AINV(K,J)+SUM
      C(I,J)=SUM
      7 CONTINUE
      15 FORMAT(/5X,27HSTIFFNESS-MATRIX IN EQ. 12.,19X,1HR//)
      PUNCH 15
160 PUNCH 8,R
      8 FORMAT(52X,F12.6)
170 PUNCH 9,((C(I,J),J=1,IMB),I=1,INA)
      9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
      25 INDEX=INDEX+1
      R=R-7.
      GO TO 102
      30 STOP
      END

```

```

C      DATA OF PROGRAM 3
      3      3      3
      20449.9000      -227054.2800      14197.4900
      -227054.2800      3266355.0000      -278175.0500
      14197.4900      -278175.0500      92871.4480
      29003.9020      -286620.0200      17922.0810
      -286620.0300      3669879.6000      -312540.7200
      17922.0810      -312540.7200      104344.7600

```

```

C C PROGRAM 4 LFCU 12/4/66
C CAL. OF STIFFNESS MATRIX IN EQ.(16)
  DIMENSION A(10,10),S(10),T(10),H(10,10)
  INDEX=1
  PI=3.1415926536
  X=PI/9.
  EI=3220000.*863370./144000.
  R=200./PI
  READ,N
5 Q=R/(4.*EI)
  SR=R*Q
  CR=R*SR
  GK=14.2
  X2=2.*X
  S2=SINF(X2)
  S1=SINF(X)
  SS=S1**2
  C1=COSF(X)
  A(1,1)=CR*(X2-S2+GK*(6.*X+S2-8.*S1))
  A(1,2)=-SR*(2.*SS+2.*GK*(2.-2.*C1-SS))
  A(1,3)=SR*(S2-X2+GK*(4.*S1-X2-S2))
  A(2,1)=A(1,2)
  A(3,1)=A(1,3)
  A(2,2)=Q*(X2+S2+GK*(X2-S2))
  A(2,3)=-Q*(2.*(GK-1.)*SS)
  A(3,2)=A(2,3)
  A(3,3)=Q*(X2-S2+GK*(X2+S2))

```



```

NN=N-1
DO 6 I=1,3
DO 6 J=1,3
6 H(I,J)=A(I,J)
H(1,1)=1./H(1,1)
DO 110 M=1,NN
K=M+1
50 DO 60 I=1,M
S(I)=0
DO 60 J=1,M
60 S(I)=S(I)+H(I,J)*H(J,K)
D=0
DO 70 I=1,M
70 D=D+H(K,I)*S(I)
D=-D+H(K,K)
H(K,K)=1./D
DO 80 I=1,M
80 H(I,K)=-S(I)*H(K,K)
DO 90 J=1,M
T(J)=0
DO 90 I=1,M
90 T(J)=T(J)+H(K,I)*H(I,J)
DO 100 J=1,M
100 H(K,J)=-T(J)*H(K,K)
DO 110 I=1,M
DO 110 J=1,M
110 H(I,J)=H(I,J)-S(I)*H(K,J)

```

```

15 FORMAT(/5X,34HSTIFFNESS=MATRIX INV.(A) IN EQ. 9.,19X,1HR//)
PUNCH 15

160 PUNCH 8,R

8 FORMAT(52X,F12.6)

170 PUNCH 9,((H(I,J),J=1,N),I=1,N)

9 FORMAT(3F16.4)

IF(INDEX-2) 25,30,30

25 INDEX=INDEX+1

R=R-7.

GO TO 5

30 STOP

END

C DATA OF PROGRAM 4

3

C C PROGRAM 5 LECU 12/4/66

C CAL. OF STIFFNESS MATRIX IN EQ. (18)

DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3)

INDEX=1

PI=3.1415926536

X=PI/9.

R=200./PI

101 READ 1,INA,IMA,IMB

1 FORMAT(3I8)

102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)

```

```

48 FORMAT(3F16.4)
5 S1=SINF(X)
  C1=COSE(X)
  B(1,1)=-1.0
  B(1,2)=-0.
  B(1,3)=-0.
  R(2,1)=+R*S1
  B(2,2)=-C1
  B(2,3)=-S1
  B(3,1)=-R*(1.-C1)
  B(3,2)=+S1
  B(3,3)=-C1
  DO 121 I=1,3
  DO 121 J=1,3
121 BQ(I,J)=B(I,J)
  DO 7 I=1,INA
  DO 7 J=1,IMB
  SUM=0.
  DO 6 K=1,IMA
6 SUM=BQ(I,K)*AINV(K,J)+SUM
  C(I,J)=SUM
7 CONTINUE
15 FORMAT(/5X,27HSTIFFNESS-MATRIX IN EQ. 12.,19X,1HR//)
  PUNCH 15
160 PUNCH 8,R
  8 FORMAT(52X,F12.6)
170 PUNCH 9,((C(I,J),J=1,IMB),I=1,INA)

```

```

9  FORMAT(3F16.4)
   IF(INDEX-2) 25,30,30
25  INDEX=INDEX+1
   R=R-7.
   GO TO 102
30  STOP
   END

```

```

C    DATA OF PROGRAM 5
      3      3      3
20449.9000    227054.2800    14197.4900
227054.2800    3266355.0000    278175.0500
14197.4900    278175.0500    92871.4480
29003.9020    286620.0200    17922.0810
286620.0200    3669879.6000    312540.7200
17922.0810    312540.7200    104344.7600

```

```

C  C  PROGRAM 6  LEOU  11/26/66
C    CAL. OF STIFFNESS MATRICES OF STRAIGHT BEAM. IN EQ.(59),(61)
      DIMENSION F(3,3),G(3,3)
      EI=3220000.*228750./144000.
      SL=7.
      GJ=3220000.*47000./(2.25*144000.)
      F(1,1)=12.*EI/(SL**3)
      F(1,2)=6.*EI/(SL**2)
      F(1,3)=0.
      F(2,1)=F(1,2)
      F(2,2)=4.*EI/SL
      F(2,3)=0.

```

```

F(3,1)=0.
F(3,2)=0.
F(3,3)=GJ/SL
G(1,1)=-12.*EI/(SL**3)
G(1,2)=-6.*EI/(SL**2)
G(1,3)=-0.
G(2,1)=-G(1,2)
G(2,2)=+2.*EI/SL
G(2,3)=-0.
G(3,1)=-0.
G(3,2)=-0.
G(3,3)=-GJ/SL
1) FORMAT(/10X,29HSTIFFNESS-MATRIX FOR NEAR END/)
PUNCH 10
16 FORMAT(3F16.4)
PUNCH 16,((F(I,J),J=1,3),I=1,3)
2) FORMAT(/9X,28HSTIFFNESS-MATRIX FOR FAR END/)
PUNCH 21
PUNCH 16,((G(I,J),J=1,3),I=1,3)
STOP
END

```

```

C C PROGRAM 7 LFCU 11/26/66
C CAL. OF STIFFNESS MATRICES OF STRAIGHT BEAM. IN EQ.(66),(68)
  DIMENSION F(3,3),G(3,3)
  EI=3220000.*228750./144000.
  SL=7.
  GJ=3220000.*47000./(2.25*144000.)
  F(1,1)=12.*EI/(SL**3)
  F(1,2)=-6.*EI/(SL**2)
  F(1,3)=0.
  F(2,1)=F(1,2)
  F(2,2)=4.*EI/SL
  F(2,3)=0.
  F(3,1)=0.
  F(3,2)=0.
  F(3,3)=GJ/SL
  G(1,1)=-12.*EI/(SL**3)
  G(1,2)=6.*EI/(SL**2)
  G(1,3)=-0.
  G(2,1)=-G(1,2)
  G(2,2)=+2.*EI/SL
  G(2,3)=-0.
  G(3,1)=-0.
  G(3,2)=-0.
  G(3,3)=-GJ/SL
10 FORMAT(/10X,29HSTIFFNESS-MATRIX FOR NEAR END/)
  PUNCH 10

```

```

16 FORMAT(3F16.4)
   PUNCH 16,((F(I,J),J=1,3),I=1,3)
21 FORMAT(/ /9X,28HSTIFFNESS-MATRIX FOR FAR END/)
   PUNCH 21
   PUNCH 16,((G(I,J),J=1,3),I=1,3)
   STOP
   END

```

C C PROGRAM 8 MATRIX MULTIPLICATION LECU 11/25/66

```

   DIMENSION A(12,24),B(24,12),C(12,12)
100 READ 1,INA,IMA,IMB
   1 FORMAT(3I8)
   READ 4,((A(I,J),J=1,IMA),I=1,INA)
   READ 2,((B(I,J),J=1,IMB),I=1,IMA)
   4 FORMAT(6F11.8)
   2 FORMAT(6F12.2)
   DO 7 I=1,INA
   DO 7 J=1,IMB
   SUM=0.
   DO 6 K=1,IMA
   6 SUM=A(I,K)*B(K,J)+SUM
   C(I,J)=SUM
   7 CONTINUE
   PUNCH 3,((C(I,J),J=1,IMB),I=1,INA)
   3 FORMAT(6F12.2)

```

STOP

END

C DATA OF PROGRAM 8

12        24        12

C C PROGRAM 9 MATRIX INVERSION BY PARTITIONING LECU 11/2/66

DIMENSION A(20,20),B(19),C(19)

4 FORMAT(6F12.2)

RFAD,N

NN=N-1

10 READ 4,((A(I,J),J=1,N),I=1,N)

A(1,1)=1./A(1,1)

DO 110 M=1,NN

K=M+1

50 DO 60 I=1,M

B(I)=0

DO 60 J=1,M

60 B(I)=B(I)+A(I,J)\*A(J,K)

D=0

DO 70 I=1,M

70 D=D+A(K,I)\*B(I)

D=-D+A(K,K)

A(K,K)=1./D

DO 80 I=1,M

80 A(I,K)=-B(I)\*A(K,K)



```

      DO 90 J=1,M
      C(J)=0
      DO 90 I=1,M
90    C(J)=C(J)+A(K,I)*A(I,J)
      DO 100 J=1,M
100   A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
110   A(I,J)=A(I,J)-B(I)*A(K,J)
      DO 170 I=1,N
170   PUNCH 5,(A(I,J),J=1,N)
      5 FORMAT(4E16.8)
      END
C      DATA OF PROGRAM 9
      12

```

```

C C PROGRAM 10 MATRIX TIMES VECTOR   LECU 11/29/66
      DIMENSION A(48,12),VEC(12),PRCD(48)
100 READ 1,INA,IMA
      1 FORMAT(2I8)
      2 FORMAT(6F12.2)
      3 FORMAT(F20.15)
      READ 2,((A(I,J),J=1,IMA),I=1,INA)
      READ 3,(VEC(I),I=1,IMA)

```

```

      DO 115 I=1,INA
      SUM=0.0
      DO 116 K=1,IMA
116  SUM=A(I,K)*VEC(K)+SUM
      PRCD(I)=SUM
115  CONTINUE
      5  FORMAT(20X,32HPRODUCT OF A MATRIX AND A VECTOR//)
      PUNCH 5
      PUNCH 6, (PRCD(I), I=1, INA)
      6  FORMAT(30X,F10.5)
      STOP
      END
C      DATA OF PROGRAM 10
      48      12

```

```

C  C  PROGRAM 11   LECU  11/25/66
C      CAL. OF CONSTANTS G,H,S IN EQ.(46),(47) AND (48)
      PI=3.1415926536
      INDEX=1
      X=PI/9.
      P=1.
      GK=14.2
      R=200./PI
      5  Z=PI/36.
      DZ=PI/36.

```

```

SX=SINF(X)
X2=2.*X
SX2=SINF(X2)
CX=COSE(X)
CX2=COSE(X2)
Y=GK+1.
YN=GK-1.
10 SZ=SINF(Z)
Z2=2.*Z
CZ=COSE(Z)
SZ2=SINF(Z2)
CZ2=COSE(Z2)
XZ=X-Z
SXZ=SINF(X-Z)
EI=3220000.*863370./144000.
T=(P*R**2)/(4.*EI)
CR=P*T
20 GA=-YN*CZ*(SX2-SZ2)
GR=SZ*YN*(CX2-CZ2)
GC=-2.*XZ*(Y*CZ+2.*GK)
GD=4.*GK*SXZ+4.*GK*(SX-SZ)
G=(GA+GB+GC+GD)*CR
HA=2.*XZ*Y*SZ
HB=4.*GK*(CX-CZ)
HC=-CZ*(CX2-CZ2)*YN
HD=-SZ*(SX2-SZ2)*YN
H=T*(HA+HB+HC+HD)

```

```

SA=2.*XZ*Y*CZ
SR=-4.*GK*(SX-SZ)
SC=CZ*(SX2-SZ2)*YN
SD=-SZ*(CX2-CZ2)*YN
S=T*(SA+SR+SC+SD)
25 FORMAT(13X,1HR,F15.8)
PUNCH 25,R
30 FORMAT(//4X,1HG,15X,1HH,15X,1HS,19X,1HZ)
PUNCH 30
35 FORMAT(50X,F14.8)
PUNCH 35,Z
40 FORMAT(3F16.12)
PUNCH 40,G,H,S
IF(Z-PI/12.) 50,54,54
50 Z=Z+DZ
GO TO 10
54 CONTINUE
IF(INDEX-2) 55,60,60
55 INDEX=INDEX+1
R=R-7.
GO TO 5
60 STOP
END

```

```

C C PROGRAM 12 LECU 11/29/66
C CAL. OF FIXED-END STRESSES IN EQ.(45)
  DIMENSION A(12,12),VEC(12),PRCD(12)
  PI=3.1415926536
  DZ=PI/36.
  N=1
  R=200./PI
100 READ 1,INA,IMA
  1 FORMAT(2I8)
  2 FORMAT(3F16.4)
  3 FORMAT(3F16.12)
16 Z=PI/36.
  INDEX=1
  8 READ 2,((A(I,J),J=1,IMA),I=1,INA)
  9 READ 3,(VEC(I),I=1,IMA)
  DO 115 I=1,INA
    SUM=0.0
    DO 116 K=1,IMA
116 SUM=A(I,K)*VEC(K)+SUM
    PRCD(I)=SUM
115 CONTINUE
  4 FORMAT(/13X,1HR,F15.8)
  PUNCH 4,R
  5 FORMAT(17X,18HFIXED-END STRESSES)
  PUNCH 5
40 FORMAT(/6X,1HF,13X,1HM,13X,1HT,15X,1HZ)
  PUNCH 40

```

```

6  FORMAT(3F14.8,F14.8)
   PUNCH 6,(PRCD(I),I=1,INA),Z
   IF(INDEX-3)20,30,30
20  INDEX=INDEX+1
   Z=Z+DZ
   GO TO 9
30  IF(N-2)15,50,50
15  N=N+1
   R=R-7.
   GO TO 16
50  STOP
   END
C    DATA OF PROGRAM 12
      3      3
      -20449.9000      +227054.2800      -14197.4900
      +227054.2800      -3266355.0000      +278175.0500
      -14197.4900      +278175.0500      -92871.4480
      -.000140844240    -.000009378848    -.000006671969
      -.000066033581    -.000003845325    -.000001600150
      -.000017110796    -.000000864254    -.000000061282
      -29003.9020      +286620.0200      -17922.0810
      +286620.0300      -3669879.6000      +312540.7200
      -17922.0810      +312540.7200      -104344.7600
      -.000099305638    -.000007429725    -.000005285392
      -.000046558573    -.000003046185    -.000001267605
      -.000012064381    -.000000684644    -.000000048546

```

```

C C PROGRAM 13 LECU 12/7/66
C CAL. OF FIXED-END STRESSES IN EQ.(50)
  DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3),D(3,3),F(3),G(3),H(3)
  DIMENSION VEC(3),PRCD(3)
  INDEX=1
  P=1.
  PI=3.1415926536
  X=PI/9.
  R=200./PI
101 READ 1,INA,IMA,IMR
  1 FORMAT(3I8)
  26 Z=PI/36.
  DZ=PI/36.
102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)
  48 FORMAT(3F16.4)
  5 S1=SINF(X)
  C1=COSE(X)
  XZ=X-Z
  SXZ=SINF(XZ)
  CXZ=COSE(XZ)
  B(1,1)=+1.0
  B(1,2)=+0.
  B(1,3)=+0.
  R(2,1)=+R*S1
  B(2,2)=+C1
  B(2,3)=-S1
  B(3,1)=+R*(1.-C1)

```

```

      R(3,2)=+S1
      B(3,3)=+C1
      DO 121 I=1,3
      DO 121 J=1,3
121  BQ(I,J)=R(I,J)
      DO 7 I=1,INA
      DO 7 J=1,IMB
      SUM=0.
      DO 6 K=1,IMA
6  SUM=BQ(I,K)*AINV(K,J)+SUM
      C(I,J)=SUM
7  CONTINUE
      DO 130 I=1,3
      DO 130 J=1,3
130 D(I,J)=C(I,J)
10 READ 15,(VEC(I),I=1,IMA)
15 FORMAT(3F16.12)
      DO 115 I=1,INA
      SUM=0.0
      DO 116 K=1,IMA
116 SUM=D(I,K)*VEC(K)+SUM
      PRCD(I)=SUM
115 CONTINUE
      DO 120 I=1,3
120 F(I)=PRCD(I)
      G(1)=P
      G(2)=P*R*SXZ

```



```

      G(I)=P*R*(1.-CXZ)
      DO 135 I=1,3
135  H(I)=F(I)+G(I)
      16  FORMAT(/13X,1HR,F15.8)
      PUNCH 16,R
      17  FORMAT(17X,18HFIXED-END STRESSES)
      PUNCH 17
      41  FORMAT(/6X,1HF,13X,1HM,13X,1HT,15X,1HZ)
      PUNCH 41
      20  FORMAT(3F14.8,F14.8)
      PUNCH 20,(H(I),I=1,INA),Z
      IF(Z-PI/12.) 30,40,40
      30  Z=Z+DZ
      GO TO 5
      40  CONTINUE
      IF(INDEX-2) 45,60,60
      45  INDEX=INDEX+1
      R=R-7.
      GO TO 26
      60  STOP
      END

```

C DATA OF PROGRAM 13

3	3	3	
20449.9000	-227054.2800	14197.4900	
-227054.2800	3266355.0000	-278175.0500	
14197.4900	-278175.0500	92871.4480	
-.000140844240	-.000009378848	-.000006671969	
-.000066033581	-.000003845325	-.000001600150	
-.000017110796	-.000000864254	-.000000061282	
29003.9020	-286620.0200	17922.0810	
-286620.0300	3669879.6000	-312540.7200	
17922.0810	-312540.7200	104344.7600	
-.000099305638	-.000007429725	-.000005285392	
-.000046558573	-.000003046185	-.000001267605	
-.000012064381	-.000000684644	-.000000048546	

Table 1. Final member end stresses of the structure in Fig. 8 due to uniform load.

	$\{P_1\}$	fixed + end stresses	= final member end stresses
$F_1^1$	47.65	+ 19.778	= + 67.428 kips
$M_1^1$	-767.50	- 75.369	= - 842.829 ft-kips
$T_1^1$	56.73	- 0.347	= + 56.383 ft-kips
$F_2^1$	-47.65	+ 19.778	= - 27.872 kips
$M_2^1$	-296.85	+ 75.369	= - 221.481 ft-kips
$T_2^1$	26.26	- 0.347	= + 25.913 ft-kips
$F_2^4$	3.75	+ 4.340	= + 8.090 kips
$M_2^4$	20.64	+ 5.060	= + 25.700 ft-kips
$T_2^4$	6.10	+ 0.000	= + 6.100 ft-kips
$F_2^2$	- .00	+ 19.778	= + 19.778 kips
$M_2^2$	290.75	- 75.369	= + 215.381 ft-kips
$T_2^2$	-51.27	- 0.347	= - 51.617 ft-kips
$F_3^2$	.00	+ 19.778	= + 19.778 kips
$M_3^2$	-290.75	+ 75.370	= - 215.381 ft-kips
$T_3^2$	-51.27	- 0.347	= - 51.617 ft-kips
$F_3^5$	3.75	+ 4.340	= + 8.090 kips
$M_3^5$	20.64	+ 5.060	= + 25.700 ft-kips
$T_3^5$	-6.10	+ 0.000	= - 6.100 ft-kips
$F_3^3$	-47.65	+ 19.778	= - 27.872 kips
$M_3^3$	296.85	- 75.369	= + 221.481 ft-kips
$T_3^3$	26.26	- 0.347	= + 25.913 ft-kips
$F_4^3$	47.65	+ 19.780	= + 67.428 kips
$M_4^3$	767.50	+ 75.369	= + 842.829 ft-kips
$T_4^3$	56.73	- 0.347	= + 56.383 ft-kips

Table 2. Final member end stresses of the structure in Fig. 8 due to uniform load.

	$\{P_2\}$	fixed + end stresses	= final member end stresses
$F_5^6$	35.79	+ 17.603	= + 53.403 kips
$M_5^6$	-543.80	- 59.706	= - 603.536 ft-kips
$T_5^6$	34.56	- 0.275	= + 34.285 ft-kips
$F_6^6$	-35.79	+ 17.603	= - 18.197 kips
$M_6^6$	-170.81	+ 59.706	= - 111.144 ft-kips
$T_6^6$	31.21	- 0.275	= + 30.925 ft-kips
$F_6^4$	-3.75	+ 4.340	= + 0.570 kips
$M_6^4$	5.59	- 5.060	= + 0.550 ft-kips
$T_6^4$	-6.10	+ 0.000	= - 6.100 ft-kips
$F_6^7$	- .00	+ 17.603	= + 17.603 kips
$M_6^7$	176.92	- 59.706	= + 117.244 ft-kips
$T_6^7$	-31.19	- 0.275	= - 31.475 ft-kips
$F_7^7$	.00	+ 17.603	= + 17.603 kips
$M_7^7$	-176.92	+ 59.706	= - 117.244 ft-kips
$T_7^7$	-31.19	- 0.275	= - 31.475 ft-kips
$F_7^5$	-3.75	+ 4.340	= + 0.570 kips
$M_7^5$	5.59	- 5.060	= + 0.550 ft-kips
$T_7^5$	6.10	+ 0.000	= + 6.100 ft-kips
$F_7^8$	-35.79	+ 17.603	= - 18.197 kips
$M_7^8$	170.81	- 59.706	= + 111.144 ft-kips
$T_7^8$	31.21	- 0.275	= + 30.925 ft-kips
$F_8^8$	35.79	+ 17.603	= + 53.403 kips
$M_8^8$	543.80	+ 59.706	= + 603.536 ft-kips
$T_8^8$	34.56	- 0.275	= + 34.285 ft-kips

TABLE 3. MATRIX Q--UNIT LOAD AT POINTS 9,10,11,2

	UNIT LOAD AT POINT			
	9	10	11	2
F <sub>2</sub>	-.15453180	-.50000000	-.84544850	-1.00000000
M <sub>2</sub>	-1.07909800	-2.87811500	-3.20024310	0.00000000
T <sub>2</sub>	.00822890	.01644997	.01027509	0.00000000
F <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 4. MATRIX Q--UNIT LOAD AT POINTS 12,13,14,3

	UNIT LOAD AT POINT			
	12	13	14	3
F <sub>2</sub>	-.84546821	-.50000000	-.15455155	0.00000000
M <sub>2</sub>	3.20061630	2.87812000	1.07916620	0.00000000
T <sub>2</sub>	.01030862	.01644585	.00820781	0.00000000
F <sub>3</sub>	-.15453180	-.50000000	-.84544850	-1.00000000
M <sub>3</sub>	-1.07909800	-2.87812000	-3.20024310	0.00000000
T <sub>3</sub>	.00822890	.01644997	.01027509	0.00000000
F <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 5. MATRIX Q--UNIT LOAD AT POINTS 15,16,17,18

	UNIT LOAD AT POINT			
	15	16	17	18
F <sub>2</sub>	0.00000000	0.00000000	0.00000000	-.50000000
M <sub>2</sub>	0.00000000	0.00000000	0.00000000	-.87500000
T <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>3</sub>	-.84546821	-.50000000	-.15455155	0.00000000
M <sub>3</sub>	3.20061630	2.87812000	1.07916620	0.00000000
T <sub>3</sub>	.01030862	.01644585	.00820781	0.00000000
F <sub>6</sub>	0.00000000	0.00000000	0.00000000	-.50000000
M <sub>6</sub>	0.00000000	0.00000000	0.00000000	.87500000
T <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 6. MATRIX Q--UNIT LOAD AT POINTS 19,20,21,22

	UNIT LOAD AT POINT			
	19	20	21	22
F <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>3</sub>	-.50000000	0.00000000	0.00000000	0.00000000
M <sub>3</sub>	-.87500000	0.00000000	0.00000000	0.00000000
T <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>6</sub>	0.00000000	-.15453170	-.50000000	-.84544860
M <sub>6</sub>	0.00000000	-.96044500	-2.56165000	-2.84835840
T <sub>6</sub>	0.00000000	.00732360	.01464100	.00914525
F <sub>7</sub>	-.50000000	0.00000000	0.00000000	0.00000000
M <sub>7</sub>	.87500000	0.00000000	0.00000000	0.00000000
T <sub>7</sub>	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 7. MATRIX Q--UNIT LOAD AT POINTS 6,23,24,25

	UNIT LOAD AT POINT			
	6	23	24	25
F <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>6</sub>	-1.00000000	-.84546832	-.50000000	-.15455148
M <sub>6</sub>	0.00000000	2.84868920	2.56165000	.96050480
T <sub>6</sub>	0.00000000	.00917510	.01463761	.00730520
F <sub>7</sub>	0.00000000	-.15453170	-.50000000	-.84544860
M <sub>7</sub>	0.00000000	-.96044500	-2.56165000	-2.84835840
T <sub>7</sub>	0.00000000	.00732360	.01464100	.00914525

TABLE 8. MATRIX Q--UNIT LOAD AT POINTS 7,26,27,28

	UNIT LOAD AT POINT			
	7	26	27	28
F <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>2</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>3</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
M <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
T <sub>6</sub>	0.00000000	0.00000000	0.00000000	0.00000000
F <sub>7</sub>	-1.00000000	-.84546832	-.50000000	-.15455148
M <sub>7</sub>	0.00000000	2.84868920	2.56165000	.96050480
T <sub>7</sub>	0.00000000	.00917510	.01463761	.00730520

TABLE 9. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 9,10,11,2,12,13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F <sub>1</sub> <sup>1</sup>	.97255	.89568	.78215	.64993	.51628	.38767
M <sub>1</sub> <sup>1</sup>	-4.74157	-7.83342	-9.38622	-9.69314	-9.13661	-7.90639
T <sub>1</sub> <sup>1</sup>	.05070	.19010	.36607	.49988	.54712	.52594
F <sub>2</sub> <sup>1</sup>	.02745	.10432	.21785	.35008	-.51628	-.38767
M <sub>2</sub> <sup>1</sup>	-.21906	-1.01026	-2.53080	-4.87174	-2.46867	-.83152
T <sub>2</sub> <sup>1</sup>	-.02643	-.03488	.06966	.35026	.62862	.72155
F <sub>2</sub> <sup>4</sup>	-.01223	-.04269	-.07975	-.10904	-.11888	-.11233
M <sub>2</sub> <sup>4</sup>	.01797	.04569	.05203	.02768	-.01683	-.08219
T <sub>2</sub> <sup>4</sup>	-.00299	.00009	.02681	.09277	.15780	.17696
F <sub>2</sub> <sup>2</sup>	-.01522	-.06162	-.13810	-.24104	.63516	.50000
M <sub>2</sub> <sup>2</sup>	.22205	1.01018	2.50399	4.77897	2.31087	.65457
T <sub>2</sub> <sup>2</sup>	.00846	-.01081	-.12168	-.37793	-.61178	-.63936
F <sub>3</sub> <sup>2</sup>	.01522	.06162	.13810	.24104	.36484	.50000
M <sub>3</sub> <sup>2</sup>	.12558	.38881	.61230	.62823	.27344	-.65457
T <sub>3</sub> <sup>2</sup>	-.02547	-.09875	-.21187	-.35396	-.52064	-.63936
F <sub>3</sub> <sup>5</sup>	-.00565	-.02107	-.04339	-.06942	-.09436	-.11233
M <sub>3</sub> <sup>5</sup>	-.02362	-.07759	-.13472	-.16840	-.14626	-.08218
T <sub>3</sub> <sup>5</sup>	-.01124	-.04070	-.08081	-.12354	-.16056	-.17696
F <sub>3</sub> <sup>3</sup>	-.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M <sub>3</sub> <sup>3</sup>	-.11434	-.34812	-.53149	-.50469	-.11288	.83152
T <sub>3</sub> <sup>3</sup>	.04909	.17634	.34660	.52236	.66691	.72155
F <sub>4</sub> <sup>3</sup>	.00957	.04055	.09471	.17161	.27048	.38767
M <sub>4</sub> <sup>3</sup>	.33252	1.27032	2.68008	4.38952	6.22349	7.90638
T <sub>4</sub> <sup>3</sup>	.02970	.10903	.21969	.34062	.45036	.52594

CONTINUATION OF TABLE 9

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F <sub>5</sub> <sup>6</sup>	.01234	.04271	.07911	.10757	.11772	.11233
M <sub>5</sub> <sup>6</sup>	-.29205	-1.00995	-1.87921	-2.59851	-2.93084	-2.87971
T <sub>5</sub> <sup>6</sup>	-.01240	-.03935	-.06534	-.07698	-.06906	-.04656
F <sub>6</sub> <sup>6</sup>	-.01234	-.04271	-.07911	-.10757	-.11772	-.11233
M <sub>6</sub> <sup>6</sup>	.03098	.10797	.21039	.33076	.44903	.51323
T <sub>6</sub> <sup>6</sup>	.06936	.23647	.43380	.59349	.66503	.64482
F <sub>6</sub> <sup>4</sup>	.01223	.04269	.07975	.10904	.11888	.11233
M <sub>6</sub> <sup>4</sup>	-.10359	-.34453	-.61028	-.79093	-.81530	-.70412
T <sub>6</sub> <sup>4</sup>	.00299	-.00009	-.02681	-.09277	-.15780	-.17696
F <sub>6</sub> <sup>7</sup>	.00011	.00001	-.00064	-.00146	-.00115	-0.00000
M <sub>6</sub> <sup>7</sup>	-.03296	-.10788	-.18358	-.23799	-.29123	-.33627
T <sub>6</sub> <sup>7</sup>	.03423	.10806	.17648	.19744	.15027	.05930
F <sub>7</sub> <sup>7</sup>	-.00011	-.00001	.00064	.00146	.00115	0.00000
M <sub>7</sub> <sup>7</sup>	.04144	.13808	.24527	.31952	.34737	.33628
T <sub>7</sub> <sup>7</sup>	-.02093	-.06470	-.10087	-.09913	-.03767	.05930
F <sub>7</sub> <sup>5</sup>	.00565	.02107	.04339	.06942	.09436	.11233
M <sub>7</sub> <sup>5</sup>	-.01594	-.06994	-.16901	-.31757	-.51424	-.70412
T <sub>7</sub> <sup>5</sup>	.01124	.04070	.08081	.12354	.16056	.17696
F <sub>7</sub> <sup>8</sup>	-.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M <sub>7</sub> <sup>8</sup>	-.05268	-.17877	-.32608	-.44306	-.50793	-.51323
T <sub>7</sub> <sup>8</sup>	.03687	.13463	.26988	.41670	.55191	.64482
F <sub>8</sub> <sup>8</sup>	.00554	.02106	.04403	.07089	.09551	.11233
M <sub>8</sub> <sup>8</sup>	.16945	.62220	1.25200	1.93260	2.51697	2.87970
T <sub>8</sub> <sup>8</sup>	.00230	.00660	.00838	.00219	-.01854	-.04656



TABLE 10. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 14,3,15,16,17,18

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
$F_1^1$	.27048	.17161	.09470	.04055	.00957	.44215
$M_1^1$	-6.22345	-4.38952	-2.68006	-1.27032	-.33258	-6.03375
$T_1^1$	.45036	.34062	.21969	.10903	.02971	.39245
$F_2^1$	-.27048	-.17161	-.09470	-.04055	-.00957	-.44215
$M_2^1$	.11281	.50469	.53152	.34812	.11434	-3.82314
$T_2^1$	.66689	.52236	.34660	.17634	.04910	-.00266
$F_2^2$	-.09436	-.06942	-.04339	-.02108	-.00565	.56592
$M_2^4$	-.14628	-.16840	-.13473	-.07759	-.02362	1.19219
$T_2^2$	.16056	.12354	.08081	.04070	.01124	.00405
$F_2^3$	.36484	.24104	.13810	.06162	.01522	-.12377
$M_2^2$	-.27337	-.62822	-.61232	-.38881	-.12559	2.94409
$T_2^2$	-.52061	-.35396	-.21187	-.09875	-.02548	-.31453
$F_3^2$	.63516	-.24104	-.13810	-.06162	-.01522	.12377
$M_3^2$	-2.31095	-4.77896	-2.50391	-1.01017	-.22210	-.17917
$T_3^2$	-.61173	-.37793	-.12166	-.01081	.00846	-.23618
$F_3^3$	-.11888	-.10903	-.07975	-.04269	-.01224	.01044
$M_3^5$	-.01685	.02768	.05206	.04569	.01795	.04690
$T_3^5$	-.15779	-.09277	-.02680	-.00009	.00298	-.02532
$F_3^3$	-.51628	.35007	.21785	.10432	.02745	-.13421
$M_3^3$	2.46874	4.87174	2.53071	1.01026	.21912	.20448
$T_3^3$	.62859	.35026	.06961	-.03488	-.02641	.18929
$F_4^3$	.51628	.64993	.78215	.89568	.97255	.13421
$M_4^3$	9.13649	9.69313	9.38639	7.83341	4.74136	2.79480
$T_4^3$	.54711	.49988	.36606	.19010	.05073	.26745

CONTINUATION OF TABLE 10

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
F <sub>5</sub> <sup>6</sup>	.09551	.07089	.04403	.02106	.00554	.30251
M <sub>5</sub> <sup>6</sup>	-2.51695	-1.93260	-1.25199	-.62220	-.16949	-4.18353
T <sub>5</sub> <sup>6</sup>	-.01853	.00219	.00838	.00660	.00230	.27819
F <sub>6</sub> <sup>6</sup>	-.09551	-.07089	-.04403	-.02106	-.00554	-.30251
M <sub>6</sub> <sup>6</sup>	.50790	.44306	.32609	.17877	.05269	-1.83615
T <sub>6</sub> <sup>6</sup>	.55190	.41670	.26988	.13463	.03688	.13571
F <sub>6</sub> <sup>4</sup>	.09436	.06942	.04339	.02108	.00565	.43408
M <sub>6</sub> <sup>4</sup>	-.51424	-.31757	-.16902	-.06994	-.01594	-.73074
T <sub>6</sub> <sup>4</sup>	-.16056	-.12354	-.08081	-.04070	-.01124	-.00405
F <sub>6</sub> <sup>7</sup>	.00115	.00146	.00064	-.00001	-.00011	-.13157
M <sub>6</sub> <sup>7</sup>	-.34734	-.31952	-.24528	-.13808	-.04145	2.71521
T <sub>6</sub> <sup>7</sup>	-.03767	-.09913	-.10087	-.06470	-.02094	-.27997
F <sub>7</sub> <sup>7</sup>	-.00115	-.00146	-.00064	.00001	.00011	.13157
M <sub>7</sub> <sup>7</sup>	.29121	.23800	.18361	.10788	.03395	-.09752
T <sub>7</sub> <sup>7</sup>	.15026	.19744	.17649	.10806	.03423	-.21598
F <sub>7</sub> <sup>5</sup>	.11888	.10903	.07975	.04269	.01224	-.01044
M <sub>7</sub> <sup>5</sup>	-.81527	-.79093	-.61030	-.34453	-.10360	.02618
T <sub>7</sub> <sup>5</sup>	.15779	.09277	.02680	.00009	-.00298	.02532
F <sub>7</sub> <sup>8</sup>	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M <sub>7</sub> <sup>8</sup>	-.44900	-.33077	-.21041	-.10796	-.03097	.07220
T <sub>7</sub> <sup>8</sup>	.66501	.59349	.43380	.23647	.06937	.18981
F <sub>8</sub> <sup>8</sup>	.11772	.10757	.07911	.04271	.01235	.12113
M <sub>8</sub> <sup>8</sup>	2.93082	2.59851	1.87921	1.00995	.29211	2.34446
T <sub>8</sub> <sup>8</sup>	-.06906	-.07698	-.06535	-.03935	-.01240	.21085

TABLE 11. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 19,20,21,22,6,23

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F <sub>1</sub> <sup>1</sup>	.14659	.02236	.07521	.13555	.18094	.19691
M <sub>1</sub> <sup>1</sup>	-3.17143	-.25630	-.86872	-1.58509	-2.16366	-2.44253
T <sub>1</sub> <sup>1</sup>	.27938	.03364	.11506	.21176	.29050	.32618
F <sub>2</sub> <sup>1</sup>	-.14659	-.02236	-.07521	-.13555	-.18094	-.19691
M <sub>2</sub> <sup>1</sup>	-.11600	-.23453	-.78202	-1.38944	-1.80724	-1.88061
T <sub>2</sub> <sup>1</sup>	.25937	-.02980	-.09977	-.17726	-.22766	-.22710
F <sub>2</sub> <sup>4</sup>	-.01527	.03057	.10089	.17681	.22553	.22558
M <sub>2</sub> <sup>4</sup>	-.00339	.07292	.24461	.43692	.56892	.58197
T <sub>2</sub> <sup>4</sup>	.04646	.00640	.01391	.00520	-.03504	-.07639
F <sub>2</sub> <sup>2</sup>	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M <sub>2</sub> <sup>2</sup>	.06955	.22813	.76811	1.38424	1.84228	1.95700
T <sub>2</sub> <sup>2</sup>	-.25598	-.04311	-.14484	-.25966	-.34126	-.35487
F <sub>3</sub> <sup>2</sup>	-.16186	.00821	.02568	.04126	.04459	.02867
M <sub>3</sub> <sup>2</sup>	-3.67716	-.05037	-.21227	-.49120	-.87693	-1.33601
T <sub>3</sub> <sup>2</sup>	-.40466	-.00599	-.02803	-.07103	-.13821	-.22577
F <sub>3</sub> <sup>5</sup>	.55058	.00039	.00679	.02638	.06459	.12302
M <sub>3</sub> <sup>5</sup>	1.15441	.00712	.03791	.10392	.21191	.35575
T <sub>3</sub> <sup>5</sup>	-.05368	.00340	.01372	.03040	.05176	.07367
F <sub>3</sub> <sup>3</sup>	-.28872	-.00860	-.03247	-.06764	-.10918	-.15169
M <sub>3</sub> <sup>3</sup>	2.85584	.04697	.19855	.46080	.82517	1.26234
T <sub>3</sub> <sup>3</sup>	.12526	-.00113	-.00988	-.03289	-.07370	-.12997
F <sub>4</sub> <sup>3</sup>	.38872	.00860	.03247	.06764	.10918	.15169
M <sub>4</sub> <sup>3</sup>	5.82304	.14267	.51705	1.02848	1.57670	2.07220
T <sub>4</sub> <sup>3</sup>	.39794	.01800	.06604	.13299	.20621	.27277

CONTINUATION OF TABLE 11

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F <sub>5</sub> <sup>6</sup>	.10553	.95837	.85010	.70468	.55410	.42294
M <sub>5</sub> <sup>6</sup>	-2.18641	-3.90090	-5.90906	-6.46161	-6.13498	-5.46658
T <sub>5</sub> <sup>6</sup>	.15678	.06375	.23299	.44213	.59874	.64469
F <sub>6</sub> <sup>6</sup>	-.10553	.04163	.14990	.29532	.44589	-.42294
M <sub>6</sub> <sup>6</sup>	.06305	-.21382	-.99286	-2.48978	-4.76845	-2.83897
T <sub>6</sub> <sup>6</sup>	.23986	-.10177	-.29878	-.42985	-.35779	-.18137
F <sub>6</sub> <sup>4</sup>	.01527	-.03057	-.10089	-.17681	-.22553	-.22558
M <sub>6</sub> <sup>4</sup>	-.10352	.14108	.46162	.80073	1.00982	.99710
T <sub>6</sub> <sup>4</sup>	-.04646	-.00640	-.01391	-.00520	.03504	.07639
F <sub>6</sub> <sup>7</sup>	.09026	-.01106	-.04901	-.11851	-.22036	.64952
M <sub>6</sub> <sup>7</sup>	-.01659	.22022	1.00677	2.49498	4.73341	2.76258
T <sub>6</sub> <sup>7</sup>	-.13634	-.03930	-.16284	-.37088	-.65204	-.81573
F <sub>7</sub> <sup>7</sup>	-.09026	.01106	.04901	.11851	.22036	.35148
M <sub>7</sub> <sup>7</sup>	-1.78021	-.00604	-.05194	-.17462	-.40045	-.77154
T <sub>7</sub> <sup>7</sup>	-.17463	-.00059	-.02384	-.09984	-.25319	-.49558
F <sub>7</sub> <sup>5</sup>	.44942	-.00039	-.00679	-.02638	-.06459	-.12302
M <sub>7</sub> <sup>5</sup>	-.80036	-.00441	.00966	.08073	.24023	.50539
T <sub>7</sub> <sup>5</sup>	.05368	-.00340	-.01372	-.03040	-.05176	-.07367
F <sub>7</sub> <sup>8</sup>	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M <sub>7</sub> <sup>8</sup>	2.60153	.00944	.06566	.20502	.45221	.84522
T <sub>7</sub> <sup>8</sup>	.09999	.00500	.01418	.01910	.01297	-.00980
F <sub>8</sub> <sup>8</sup>	.35916	.01067	.04222	.09213	.15577	.22846
M <sub>8</sub> <sup>8</sup>	4.54996	.19966	.76128	1.59939	2.59827	3.62984
T <sub>8</sub> <sup>8</sup>	.24357	.02854	.10847	.22676	.36544	.50081

TABLE 12. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 24,25,7,26,27,28

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
$F_1^1$	.18465	.15169	.10918	.06764	.03247	.00860
$M_1^1$	-2.39595	-2.07215	-1.57670	-1.02849	-.51705	-.14269
$T_1^1$	.31731	.27277	.20621	.13299	.06604	.01801
$F_2^1$	-.18465	-.15169	-.10918	-.06764	-.03247	-.00860
$M_2^1$	-1.66060	-1.26234	-.82518	-.46078	-.19855	-.04699
$T_2^1$	-.18765	-.12997	-.07370	-.03288	-.00988	-.00113
$F_2^2$	.18465	.12302	.06459	.02638	.00679	.00039
$M_2^2$	.49559	.35574	.21190	.10391	.03790	.00713
$T_2^2$	-.08580	-.07367	-.05176	-.03040	-.01372	-.00340
$F_2^2$	-0.00000	.02867	.04459	.04126	.02568	.00821
$M_2^2$	1.74640	1.33601	.87694	.49119	.21227	.05038
$T_2^2$	-.30794	-.22577	-.13821	-.07103	-.02803	-.00599
$F_3^2$	0.00000	-.02867	-.04459	-.04126	-.02568	-.00821
$M_3^2$	-1.74640	-1.95694	-1.84227	-1.38427	-.76811	-.22815
$T_3^2$	-.30794	-.35487	-.34126	-.25966	-.14484	-.04312
$F_3^5$	.18466	.22558	.22554	.17681	.10089	.03057
$M_3^5$	.49560	.58197	.56892	.43693	.24462	.07293
$T_3^5$	.08580	.07639	.03504	-.00520	-.01391	-.00640
$F_3^3$	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
$M_3^3$	1.66060	1.88056	1.80724	1.38947	.78202	.23455
$T_3^3$	-.18765	-.22709	-.22766	-.17726	-.09978	-.02981
$F_4^3$	.18465	.19690	.18094	.13555	.07521	.02236
$M_4^3$	2.39595	2.44246	2.16365	1.58513	.86871	.25632
$T_4^3$	.31731	.32617	.29050	.21176	.11506	.03365

CONTINUATION OF TABLE 12

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
F <sub>5</sub> <sup>6</sup>	.31534	.22846	.15577	.09213	.04222	.01067
M <sub>5</sub> <sup>6</sup>	-4.60897	-3.62984	-2.59827	-1.59937	-.76128	-.19970
T <sub>5</sub> <sup>6</sup>	.60347	.50081	.36544	.22676	.10847	.02855
F <sub>6</sub> <sup>6</sup>	-.31534	-.22846	-.15577	-.09213	-.04222	-.01067
M <sub>6</sub> <sup>6</sup>	-1.57382	-.84527	-.45221	-.20501	-.06566	-.00945
T <sub>6</sub> <sup>6</sup>	-.06829	-.00981	.01297	.01911	.01418	.00500
F <sub>6</sub> <sup>4</sup>	-.18465	-.12302	-.06459	-.02638	-.00679	-.00039
M <sub>6</sub> <sup>4</sup>	.79699	.50537	.24022	.08072	.00966	-.00440
T <sub>6</sub> <sup>4</sup>	.08580	.07367	.05176	.03040	.01372	.00340
F <sub>6</sub> <sup>7</sup>	.50000	.34148	.22036	.11851	.04901	.01106
M <sub>6</sub> <sup>7</sup>	1.48802	.77161	.40045	.17461	.05194	.00605
T <sub>6</sub> <sup>7</sup>	-.72870	-.49557	-.25319	-.09983	-.02384	-.00060
F <sub>7</sub> <sup>7</sup>	.50000	.64852	-.22036	-.11851	-.04901	-.01106
M <sub>7</sub> <sup>7</sup>	-1.48802	-2.76267	-4.73341	-2.49487	-1.00677	-.22030
T <sub>7</sub> <sup>7</sup>	-.72870	-.80568	-.65204	-.37087	-.16284	-.03932
F <sub>7</sub> <sup>5</sup>	-.18466	-.22558	-.22554	-.17681	-.10089	-.03057
M <sub>7</sub> <sup>5</sup>	.79699	.99706	1.00983	.80076	.46162	.14108
T <sub>7</sub> <sup>5</sup>	-.08580	-.07639	-.03504	.00520	.01391	.00640
F <sub>7</sub> <sup>8</sup>	-.31534	-.42294	.44590	.29532	.14990	.04164
M <sub>7</sub> <sup>8</sup>	1.57382	2.83906	4.76845	2.48967	.99286	.21390
T <sub>7</sub> <sup>8</sup>	-.06829	-.18138	-.35779	-.42989	-.29878	-.10177
F <sub>8</sub> <sup>8</sup>	.31534	.42294	.55410	.70468	.85010	.95836
M <sub>8</sub> <sup>8</sup>	4.60897	5.46658	6.13497	6.46170	5.90906	3.90071
T <sub>8</sub> <sup>8</sup>	.60347	.64469	.59874	.44212	.23299	.06379

TABLE 12. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 9,10,11,2,12,13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F <sub>9</sub>	.02745	.89568	.78215	.64993	.51628	.38767
M <sub>9</sub>	-.67272	-2.85049	-5.04264	-6.09368	-6.28494	-5.77115
T <sub>9</sub>	.09071	-.27636	-.26390	-.18938	-.12620	-.07123
F <sub>10</sub>	.02745	.89568	.78215	.64993	.51628	.38767
M <sub>10</sub>	-.52378	2.15413	-.66069	-2.44785	-3.38545	-3.59199
T <sub>10</sub>	.03856	-.30677	-.51292	-.56232	-.54842	-.48004
F <sub>11</sub>	.02745	.10432	.78215	.64993	.51628	.38767
M <sub>11</sub>	-.37284	-1.58828	3.72629	1.21662	-.46018	-1.38550
T <sub>11</sub>	-.00059	.07858	-.37907	-.61607	-.71632	-.69736
F <sub>12</sub>	.01522	-.06162	-.13810	-.24104	.36484	.50000
M <sub>12</sub>	-.13607	.66538	1.73882	3.45631	-5.88207	3.48206
T <sub>12</sub>	-.02409	.06235	.06356	-.01837	.21775	-.45875
F <sub>13</sub>	.01522	-.06162	-.13810	-.24104	.36484	.50000
M <sub>13</sub>	-.04900	.31551	.96041	2.10735	-3.85435	6.28304
T <sub>13</sub>	-.03217	.10517	.18142	.22454	-.20735	-.03240
F <sub>14</sub>	.01522	-.06162	-.13810	-.24104	.36484	.50000
M <sub>14</sub>	.03843	-.03675	.17470	.74235	-1.79729	-3.48206
T <sub>14</sub>	-.03263	.11734	.23098	.34896	-.45410	-.45875
F <sub>15</sub>	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M <sub>15</sub>	.17119	-.58716	-1.08517	-1.50048	-1.67134	-1.38552

CONTINUATION OF TABLE 13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
T <sub>15</sub>	-.03661	.13550	.27601	.43481	.58901	.69736
F <sub>16</sub>	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M <sub>16</sub>	.22683	-.82172	-1.63060	-2.48484	-3.21707	-3.59202
T <sub>16</sub>	-.01924	.07399	.15744	.26081	.37558	.48004
F <sub>17</sub>	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M <sub>17</sub>	.28074	-1.05004	-2.16362	-3.45030	-4.73832	-5.77118
T <sub>17</sub>	.00292	-.00773	-.00822	.00168	.02824	.07123
F <sub>18</sub>	-.01223	-.04269	-.07975	-.10904	-.11888	-.11233
M <sub>18</sub>	.06078	.19511	.33116	.40932	.39925	.31097
T <sub>18</sub>	-.00299	.00009	.02681	.09277	.15780	.17696
F <sub>19</sub>	-.00565	-.02107	-.04339	-.06942	-.09436	-.11233
M <sub>19</sub>	-.00385	-.00385	.01715	.07457	.18400	.31098
T <sub>19</sub>	-.01124	0.00000	-.08081	-.12354	-.16056	-.17696
F <sub>20</sub>	-.01234	.04271	.07911	.10757	.11772	.11233
M <sub>20</sub>	.22884	-.79176	-1.47569	-2.05069	-2.33232	-2.30996
T <sub>20</sub>	.03515	-.11801	-.21182	-.27997	-.29885	-.27315
F <sub>21</sub>	-.01234	.04271	.07911	.10757	.11772	.11233
M <sub>21</sub>	.16397	-.56754	-1.06093	-1.48726	-1.71605	-1.72263
T <sub>21</sub>	.05230	-.17736	-.32257	-.43444	-.47561	-.44921
F <sub>22</sub>	-.01234	.04271	.07911	.10757	.11772	.11233



CONTINUATION OF TABLE 13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
M <sub>22</sub>	.09785	-.33900	-.63810	-.91251	-1.08671	-1.12219
T <sub>22</sub>	.06374	-.21694	-.39675	-.53921	-.59798	-.57342
F <sub>23</sub>	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M <sub>23</sub>	.03622	-.11684	-.20142	-.97848	-.30890	-.34016
T <sub>23</sub>	-.03115	.09825	.15967	-.03556	.12407	.02977
F <sub>24</sub>	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M <sub>24</sub>	.03826	-.12491	-.21773	-2.14695	-.32421	-.34146
T <sub>24</sub>	-.02790	.08769	.14137	-.17202	.09643	0.00000
F <sub>25</sub>	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M <sub>25</sub>	.04000	-.13203	-.23239	-3.29908	-.33706	-.34016
T <sub>25</sub>	-.02449	.07648	.12172	-.40980	.06755	-.02975
F <sub>26</sub>	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M <sub>26</sub>	.08303	-.29383	-.56580	-.82778	-1.02577	-1.12221
T <sub>26</sub>	-.03094	.11400	.23094	.36121	.48495	.57341
F <sub>27</sub>	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M <sub>27</sub>	.11277	-.40665	-.80121	-1.20619	-1.53580	-1.72265
T <sub>27</sub>	-.02239	.08341	.17125	.27241	.37311	.44921
F <sub>28</sub>	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M <sub>28</sub>	.14165	-.51637	-1.03053	-1.57543	-2.03414	-2.30998
T <sub>28</sub>	-.01128	.04311	.09128	.15096	.21724	.27314

TABLE 14. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 14,3,15,16,17,18

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
F <sub>9</sub>	.27048	.17161	.09470	.04055	.00957	.44215
M <sub>9</sub>	-4.73826	-3.45032	-2.16357	-1.05000	-.28080	-3.59172
T <sub>9</sub>	-.02824	-.00167	.00821	.00772	.00293	-.02780
F <sub>10</sub>	.27048	.17161	.09470	.04055	.00957	.44215
M <sub>10</sub>	-3.21701	-2.48487	-1.63060	-.82168	-.22689	-1.12236
T <sub>10</sub>	-.37557	-.26081	-.15744	-.07400	-.01924	-.23363
F <sub>11</sub>	.27048	.17161	.09470	.04055	.00957	.44215
M <sub>11</sub>	-1.67127	-1.50050	-1.08523	-.58711	-.17125	1.35555
T <sub>11</sub>	-.58900	-.43482	-.27602	-.13551	-.03662	-.22345
F <sub>12</sub>	.36484	.24104	.13810	.06162	.01522	-.12377
M <sub>12</sub>	1.79736	.74243	.17472	-.03682	-.03844	2.27356
T <sub>12</sub>	-.45407	-.34897	-.23097	-.11733	-.03264	-.08672
F <sub>13</sub>	.36484	.24104	.13810	.06162	.01522	-.12377
M <sub>13</sub>	3.85441	2.10743	.96044	.31544	.04900	1.58573
T <sub>13</sub>	-.20731	-.22454	-.18141	-.10517	-.03218	.08178
F <sub>14</sub>	.36484	.24104	.13810	.06162	.01522	-.12377
M <sub>14</sub>	5.88213	3.45640	1.73884	.66531	.13606	.88583
T <sub>14</sub>	.21780	.01838	-.06356	-.06235	-.02410	.18969
F <sub>15</sub>	-.51628	.35007	.78215	.10432	.02745	-.13421
M <sub>15</sub>	-.46002	6.76504	-3.72613	1.58828	.37289	-.55746

CONTINUATION OF TABLE 14

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
T <sub>15</sub>	.71629	.85833	-.37913	.07858	-.00056	.17388
F <sub>16</sub>	-.51628	.35007	.78215	.10432	.02745	-.13421
M <sub>16</sub>	-3.38528	8.60685	.66085	2.15420	.52383	-1.31516
T <sub>16</sub>	.54840	1.52949	-.51296	.24198	.03859	.09212
F <sub>17</sub>	-.51628	.35007	.78215	.89568	.02745	-.13421
M <sub>17</sub>	-6.28478	10.38317	5.04281	2.85048	.67078	-2.06285
T <sub>17</sub>	.12620	2.35861	-.26393	-.27636	.09075	-.05537
F <sub>18</sub>	-.09436	-.06942	-.04339	-.02108	-.00565	.56592
M <sub>18</sub>	.18398	.07457	.01714	-.00381	-.00385	-.78853
T <sub>18</sub>	.16056	.12354	.08081	.04070	.01124	.00405
F <sub>19</sub>	-.11888	-.10903	-.07975	-.04269	-.01224	.01044
M <sub>19</sub>	.39923	.40929	.33119	.19511	.06079	.01036
T <sub>19</sub>	-.15779	-.09277	-.02680	-.00009	.00298	-.02532
F <sub>20</sub>	.09551	.07089	.04403	.02106	.00554	.30251
M <sub>20</sub>	-2.03409	-1.57535	-1.03052	-.51640	-.14169	-2.69794
T <sub>20</sub>	-.21723	-.15097	-.09128	-.04311	-.01129	-.02226
F <sub>21</sub>	.09551	.07089	.04403	.02106	.00554	.30251
M <sub>21</sub>	-1.53575	-1.20612	-.80120	-.40668	-.11281	-1.19181
T <sub>21</sub>	-.37309	-.27241	-.17125	-.08342	-.02240	-.19209
F <sub>22</sub>	.09551	.07089	.04403	.02106	.00554	.30251

CONTINUATION OF TABLE 14

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
M <sub>22</sub>	-1.02572	-.82770	-.56579	-.29386	-.08306	.32339
T <sub>22</sub>	-.48493	-.36121	-.23094	-.11400	-.03095	-.23001
F <sub>23</sub>	.00115	.00146	.00064	-.00001	-.00011	-.13157
M <sub>23</sub>	-.33706	-.30245	-.23239	-.13196	-.04001	2.07953
T <sub>23</sub>	-.06755	-.12629	-.12173	-.07649	-.02450	-.07063
F <sub>24</sub>	.00115	.00146	.00064	-.00001	-.00011	-.13157
M <sub>24</sub>	-.32421	-.28309	-.21774	-.12485	-.03827	1.42803
T <sub>24</sub>	-.09642	-.15185	-.14138	-.08770	-.02791	.08252
F <sub>25</sub>	.00115	.00146	.00064	-.00001	-.00011	-.13157
M <sub>25</sub>	-.30889	-.26156	-.20143	-.11678	-.03623	.76565
T <sub>25</sub>	-.12406	-.17563	-.15968	-.09825	-.03117	.17829
F <sub>26</sub>	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M <sub>26</sub>	-1.08660	-.91246	-.63810	-.33908	-.09789	-.54281
T <sub>26</sub>	.59796	.53921	.39675	.21695	.06374	.16926
F <sub>27</sub>	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M <sub>27</sub>	-1.71593	-1.48721	-1.06093	-.56762	-.16406	-1.15369
T <sub>27</sub>	.47560	.43444	.32257	.17736	.05231	.09519
F <sub>28</sub>	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M <sub>28</sub>	-2.33221	-2.05064	-1.47568	-.79183	-.22898	-1.75578
T <sub>28</sub>	.29886	.27997	.21182	.11801	.03515	-.03184

TABLE 15. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 19,20,21,22,6,23

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F <sub>9</sub>	.14659	.02236	.07521	.13555	.18094	.19691
M <sub>9</sub>	-2.37036	-.13419	-.45814	-.84541	-1.17680	-1.36911
T <sub>9</sub>	.03742	.01659	.05713	.10564	.14465	.15976
F <sub>10</sub>	.14659	.02236	.07521	.13555	.18094	.19691
M <sub>10</sub>	-1.55124	-.01106	-.04407	-.09930	-.18098	-.28527
T <sub>10</sub>	-.13380	.01025	.03520	.06440	.08537	.08753
F <sub>11</sub>	.14659	.02236	.07521	.13555	.18094	.19691
M <sub>11</sub>	-.72032	.11215	.37033	.64756	.81621	.80075
T <sub>11</sub>	-.23298	.01466	.04945	.08833	.11311	.11004
F <sub>12</sub>	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M <sub>12</sub>	.98968	.18547	.63533	1.17267	1.61760	1.82141
T <sub>12</sub>	-.20973	-.02505	-.08356	-.14802	-.19020	-.18990
F <sub>13</sub>	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M <sub>13</sub>	1.90227	.14139	.49770	.95218	1.38062	1.67195
T <sub>13</sub>	-.08347	-.01078	-.03410	-.05525	-.05929	-.03738
F <sub>14</sub>	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M <sub>14</sub>	2.80039	.09624	.35630	.72444	1.13312	1.50977
T <sub>14</sub>	.12185	-.00041	.00319	.01795	.05046	.10154
F <sub>15</sub>	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M <sub>15</sub>	.67724	-.00083	.01850	.08661	.22267	.42721

CONTINUATION OF TABLE 15

	UNIT LOAD AT POINT					
	10	20	21	22	6	23
T <sub>15</sub>	.27952	.00088	-.00040	-.00899	-.02795	-.05620
F <sub>16</sub>	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M <sub>16</sub>	-1.50651	-.04862	-.16170	-.28824	-.38153	-.41117
T <sub>16</sub>	.24331	-.00127	-.00666	-.01779	-.03489	-.05550
F <sub>17</sub>	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M <sub>17</sub>	-2.67880	-.09604	-.34066	-.66089	-.98282	-1.24642
T <sub>17</sub>	.01691	-.00759	-.02859	-.05923	-.09446	-.12787
F <sub>18</sub>	-.01527	.03057	.10089	.17681	.22553	.22558
M <sub>18</sub>	.05006	-.03408	-.10851	-.18192	-.22044	-.20756
T <sub>18</sub>	.04646	.00640	.01391	.00520	-.03504	-.07639
F <sub>19</sub>	.55058	.00039	.00679	.02638	.06459	.12302
M <sub>19</sub>	-.77262	.00576	.01415	.01159	-.01416	-.07482
T <sub>19</sub>	-.05368	.00240	.01372	.03040	.05176	.07367
F <sub>20</sub>	.10553	.04163	.85010	.70468	.55410	.42294
M <sub>20</sub>	-1.67060	-.84339	-1.70873	-2.99555	-3.42744	-3.41331
T <sub>20</sub>	-.01162	.03741	-.09961	.02923	.18124	.25699
F <sub>21</sub>	.10553	.04163	.85010	.70468	.55410	.42294
M <sub>21</sub>	-1.14208	-.63785	2.50460	.49330	-.69382	-1.33407
T <sub>21</sub>	-.13443	-.02726	-.06486	-.08003	.00130	.04971
F <sub>22</sub>	.10553	.04163	.14990	.70468	.55410	.42294

CONTINUATION OF TABLE 15

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
M22	-.60487	-.42746	-1.75539	3.97840	2.04509	.75533
T22	-.21070	-.07377	-.17879	.11521	.06030	.02444
F23	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M23	.44110	.16819	.77510	1.93256	3.68400	-6.02804
T23	-.11781	-.02234	-.08504	-.17757	-.28453	.39960
F24	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M24	.89543	.11488	.53753	1.35543	2.60655	-4.30417
T24	-.05945	-.00998	-.02773	-.03401	-.00988	-.05151
F25	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M25	1.34294	.06069	.29587	.76798	1.50926	-2.54755
T25	.03828	-.00232	.00866	.05870	.16982	-.35066
F26	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M26	.80923	-.04372	-.14433	-.25240	-.31990	-.28537
T26	.24891	.00350	.01075	.01703	.01875	.01464
F27	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M27	-.98922	-.09656	-.35321	-.70790	-1.08957	-1.41379
T27	.24105	-.00262	-.01098	-.02490	-.04279	-.05954
F28	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M28	-2.78015	-.14865	-.55941	-1.15801	-1.85095	-2.53145
T28	.07647	-.01333	-.05082	-.10636	-.17118	-.23180

TABLE 16. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 24,25,7,26,27,28

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
F <sub>9</sub>	.18465	.15169	.10918	.06764	.03247	.00860
M <sub>9</sub>	-1.38996	-1.24639	-.98289	-.66087	-.34068	-.09600
T <sub>9</sub>	.15201	.12788	.09446	.05923	.02859	.00759
F <sub>10</sub>	.18465	.15169	.10918	.06764	.03247	.00860
M <sub>10</sub>	-.37338	-.41113	-.38159	-.28821	-.16171	-.04858
T <sub>10</sub>	.07503	.05551	.03488	.01779	.00666	.00128
F <sub>11</sub>	.18465	.15169	.10918	.06764	.03247	.00860
M <sub>11</sub>	.64603	.42724	.22261	.08663	.01848	-.00079
T <sub>11</sub>	.08693	.05621	.02794	.00899	.00040	-.00088
F <sub>12</sub>	0.00000	.02867	.04459	.04126	.02568	.00821
M <sub>12</sub>	1.76659	1.50968	1.13306	.72444	.35639	.09626
T <sub>12</sub>	-.15456	-.10152	-.05045	-.01795	-.00320	.00041
F <sub>13</sub>	0.00000	.02367	.04459	.04126	.02568	.00821
M <sub>13</sub>	1.77334	1.67186	1.38055	.95218	.49780	.14141
T <sub>13</sub>	-0.00000	.03738	.05929	.05525	.03409	.01079
F <sub>14</sub>	0.00000	.02867	.04459	.04126	.02568	.00821
M <sub>14</sub>	1.76659	1.82131	1.61754	1.17268	.63542	.18549
T <sub>14</sub>	.15455	.18990	.19019	.14802	.08357	.02506
F <sub>15</sub>	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M <sub>15</sub>	.64610	.80069	.81626	.64753	.37044	.11219



CONTINUATION OF TABLE 16

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
T <sub>15</sub>	-.08694	-.11003	-.11312	-.08832	-.04946	-.01467
F <sub>16</sub>	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M <sub>16</sub>	-.37331	-.28526	-.18094	-.09933	-.04396	-.01102
T <sub>16</sub>	-.07503	-.08752	-.08538	-.06439	-.03521	-.01025
F <sub>17</sub>	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M <sub>17</sub>	-1.38988	-1.36905	-1.17675	-.84544	-.45803	-.13415
T <sub>17</sub>	-.15201	-.15975	-.14466	-.10564	-.05713	-.01659
F <sub>18</sub>	.18465	.12302	.06459	.02638	.00679	.00039
M <sub>18</sub>	-.15069	-.07483	-.01417	.01158	.01414	.00577
T <sub>18</sub>	-.08580	-.07367	-.05176	-.03040	-.01372	-.00340
F <sub>19</sub>	.18466	.22558	.22554	.17681	.10089	.03057
M <sub>19</sub>	-.15071	-.20756	-.22047	-.18191	-.10850	-.03407
T <sub>19</sub>	.08580	.07639	.03504	-.00520	-.01391	-.00640
F <sub>20</sub>	.31534	.22846	.15577	.09213	.04222	.01067
M <sub>20</sub>	-3.08675	-2.53144	-1.85098	-1.15807	-.55934	-.14874
T <sub>20</sub>	.26747	.23180	.17118	.10637	.05081	.01334
F <sub>21</sub>	.31534	.22846	.15577	.09213	.04222	.01067
M <sub>21</sub>	-1.54103	-1.41378	-1.08959	-.70796	-.35314	-.09664
T <sub>21</sub>	.06542	.05955	.04279	.02490	.01097	.00262
F <sub>22</sub>	.31534	.22846	.15577	.09213	.04222	.01067

CONTINUATION OF TABLE 16

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
M <sub>22</sub>	.01641	-.28536	-.31992	-.25246	-.14425	-.04381
T <sub>22</sub>	-.00115	-.01464	-.01875	-.01704	-.01075	-.00351
F <sub>23</sub>	.50000	.34148	.22036	.11851	.04901	.01106
M <sub>23</sub>	4.01508	2.49824	1.50922	.76790	.29585	.06070
T <sub>23</sub>	-.48843	-.35280	-.16981	-.05868	-.00865	.00231
F <sub>24</sub>	.50000	.34148	.22036	.11851	.04901	.01106
M <sub>24</sub>	6.51158	4.20585	2.60651	1.35534	.53751	.11488
T <sub>24</sub>	-.02882	-.06010	.00989	.03402	.02773	.00998
F <sub>25</sub>	.50000	.34148	.22036	.11851	.04901	.01106
M <sub>25</sub>	-4.01508	5.88145	3.68396	1.93247	.77508	.16820
T <sub>25</sub>	-.48843	.38032	.28453	.17757	.08504	.02234
F <sub>26</sub>	-.31534	-.42294	.44590	.70468	.14990	.04164
M <sub>26</sub>	.01650	.75541	6.98353	-3.97832	1.75539	.42759
T <sub>26</sub>	.00114	-.02444	.15531	.11518	-.17879	-.07376
F <sub>27</sub>	-.31534	-.42294	.44590	.70468	.14990	.04164
M <sub>27</sub>	-1.54094	-1.33399	9.14546	-.49322	2.50456	.63803
T <sub>27</sub>	-.06541	-.04970	.85952	-.08005	.00721	-.02724
F <sub>28</sub>	-.31534	-.42294	.44590	.70468	.85010	.04164
M <sub>28</sub>	-3.08666	-3.41323	11.23778	2.99564	1.70873	.84361
T <sub>28</sub>	-.26746	-.25697	1.74947	.02921	-.09961	.03745

THE APPLICATION OF A COMPUTER METHOD TO THE ANALYSIS OF  
A GRIDDED CIRCULAR CURVED FRAME

by

SHYI-JIUN LEOU

B. S., Taipei Institute of Technology,  
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Since a problem in the field of gridded frameworks can be reduced to the solution of a set of simultaneous linear equations, the difficulty in solving these problems involves the solution of a set of simultaneous equations. If any of the members of the framework are curved in plan, many trigonometric terms will appear in the coefficients of these equations. This makes the analyses of the problems in this field very complicated. Efforts to solve this kind of problems have been made by many authors. Most of them have tried to use the moment distribution method to avoid solving a set of simultaneous equations. However, the allowed simplifications will be much reduced if the structure is of the gridded type. Since the moment distribution method itself will become tedious if it is used to solve the problems of the gridded type.

In this report, advantage will be taken of electronic computer procedures to solve the simultaneous equations and to do the analysis work. It allows the greatest possible simplification in the computation required. The member can have any curvature in the plan, including segmental, but particular attention will be paid to the common, circular-arc shape.

Formulas are presented for frames of constant cross section that give the relationship between the end moments, end torques and shears and the corresponding rotations and displacements of the end sections. These relations have been derived from the expression for the total strain energy due to bending and torsion by means of Castigliano's theorem. From these relations the stiffness matrix of each single member can be written and the

stiffness matrix for the whole structure follows. Through computation with the electronic computer the end forces can be obtained. These end forces are combined with the fixed-end forces to obtain final end forces.